International Journal of Scientific Research and Modern Technology (IJSRMT) ijsrmt.com
Volume 5, Issue 2, 2026
DOI: https://doi.org/10.38124/ijsrmt.v5i2.1225

Bayesian Dynamic Conditional Correlation Model for Kogi State Financial Time Series Correlation Analysis

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Publication Date: 2026/02/17

Abstract

A Bayesian Dynamic Conditional Correlation (DCC) model is developed to estimate time-varying correlations among financial assets in Kogi State, Nigeria, integrating univariate GARCH(1,1) models with Bayesian estimation via Markov Chain Monte Carlo (MCMC) techniques to address parameter uncertainty and data scarcity. The model incorporates region-specific covariates, such as agricultural output, inflation, and interest rates, to capture the dynamic interplay of economic factors influencing asset correlations. It is applied to weekly price data of yam, cassava, rice, and local banking stocks from 2015 to 2024, sourced from Kogi State's local markets and financial institutions. Results reveal significant time-varying correlations, with yam–cassava prices exhibiting high correlations (0.4–0.8) and commodity–banking stock correlations peaking during economic shocks, providing valuable insights for portfolio optimization and risk management in Kogi State's volatile financial markets.

Keywords: *Bayesian DCC model, time-varying correlations, financial time series, Kogi State, GARCH, MCMC, agricultural commodity prices, banking stocks.*

I. INTRODUCTION

The Bayesian Dynamic Conditional Correlation (DCC) model estimates time-varying correlations among financial assets, combining univariate GARCH(1,1) models with Bayesian estimation via Markov Chain Monte Carlo (MCMC) to handle parameter uncertainty in volatile, data-scarce markets (Nakajima, 2018; Bello and Musa, 2021).

Recent literature underscores the inadequacy of static models like the Constant Conditional Correlation (CCC) model for dynamic markets, with Bayesian DCC models excelling in incorporating priors and addressing data limitations (Adebayo and Oluwaseun, 2020; Okafor and Ibrahim, 2022; Mensah and Aboagye, 2023).

This study applies the Bayesian DCC model to analyze correlations among Kogi State's financial assets—yam, cassava, rice prices, and local banking stocks—from 2015 to 2024, aiming to enhance portfolio optimization and risk management (Yusuf and Abdullahi, 2024; Akinola and Salami, 2023).

II. BAYESIAN DYNAMIC CONDITIONAL CORRELATION MODEL

The Bayesian Dynamic Conditional Correlation (DCC) model extends the frequentist DCC framework to estimate time-varying correlations among financial assets, incorporating Bayesian inference to handle parameter uncertainty in volatile, data-scarce markets like Kogi State, Nigeria (Nakajima, 2018; Bello and Musa, 2021). By combining univariate GARCH(1,1) models for volatility with a dynamic correlation structure, it leverages Markov Chain Monte Carlo (MCMC) techniques to estimate posterior distributions, enhancing robustness for incomplete datasets (Adebayo and Oluwaseun, 2020; Okafor and Ibrahim, 2022). This model is well-suited for analyzing Kogi State's financial assets, capturing dynamic relationships influenced by economic factors such as agricultural output and inflation (Yusuf and Abdullahi, 2024; Akinola and Salami, 2023).

Consider a multivariate return series $\mathrm{r}_t = (r_{1t}, r_{2t}, \dots, r_{nt})'$ for n assets (e.g., yam, cassava, rice prices, and banking stocks) at time t. The model is specified as:

$$\mathrm{r}_t|\mathcal{F}_{t-1} \sim N(0, \mathrm{H}_t), \quad \mathrm{H}_t = \mathrm{D}_t\mathrm{R}_t\mathrm{D}_t,$$

where H_t is the conditional covariance matrix, $\mathrm{D}_t = \mathrm{diag}\left(\sqrt{h_{1t}}, \dots, \sqrt{h_{nt}}\right)$ contains standard deviations from univariate GARCH(1,1) models, and R_t is the time-varying correlation matrix. For each asset i, the GARCH(1,1) model is:

$$r_{it} = \mu_i + \epsilon_{it}, \quad \epsilon_{it} = z_{it}\sqrt{h_{it}}, \quad z_{it} \sim N(0,1),$$
$$h_{it} = \omega_i + \alpha_i\epsilon_{i,t-1}^2 + \beta_i h_{i,t-1},$$

with $\omega_i > 0$, $\alpha_i, \beta_i \geq 0$, and $\alpha_i + \beta_i < 1$ for stationarity (Bello and Musa, 2021).

The correlation matrix R_t is derived from a positive definite matrix Q_t:

$$\mathrm{R}_t = \mathrm{diag}(\mathrm{Q}_t)^{-1/2}\mathrm{Q}_t\mathrm{diag}(\mathrm{Q}_t)^{-1/2},$$

where Q_t evolves as:

$$\mathrm{Q}_t = (1 - a - b)\bar{\mathrm{Q}} + a\mathrm{z}_{t-1}\mathrm{z}_{t-1}' + b\mathrm{Q}_{t-1},$$

with $\mathrm{z}_t = (z_{1t}, \dots, z_{nt})'$, $\bar{\mathrm{Q}}$ as the unconditional covariance of standardized residuals, and scalars $a, b \geq 0$, $a + b < 1$ for stationarity (Nakajima, 2018). Bayesian estimation assigns priors: $\mu_i \sim N(0,0.1)$, $\omega_i \sim \mathrm{Gamma}(1,0.1)$, $\alpha_i, \beta_i \sim \mathrm{Beta}(2,2)$, and $a, b \sim \mathrm{Beta}(1,5)$, reflecting market volatility patterns (Okafor and Ibrahim, 2022). The likelihood is:

$$p(\mathrm{r}_t|\mathrm{H}_t) \propto |\mathrm{H}_t|^{-1/2}\exp\left(-\frac{1}{2}\mathrm{r}_t'\mathrm{H}_t^{-1}\mathrm{r}_t\right).$$

Posterior distributions are sampled via MCMC, ensuring robust correlation estimates (Adebayo and Oluwaseun, 2020). To prove the model's applicability, consider the posterior of R_t. Given priors and the likelihood, MCMC samples converge to the true posterior under regularity conditions, as the GARCH and DCC structures ensure positive definiteness of H_t (Nakajima, 2018). For Kogi State's assets, incorporating covariates (e.g., agricultural output) in Q_t dynamics, as $\mathrm{Q}_t = \cdots + \sum_k \gamma_k \mathrm{x}_{kt}$, where x_{kt} are covariates, enhances fit, with $\gamma_k \sim N(0,0.1)$ (Yusuf and Abdullahi, 2024). This framework captures time-varying correlations, validated by empirical convergence in similar markets (Akinola and Salami, 2023).

- *Theorem 1: Positive Definiteness of the Conditional Covariance Matrix in the Bayesian DCC Model*

The Bayesian Dynamic Conditional Correlation (DCC) model is designed to capture time-varying correlations among financial assets, such as yam, cassava, rice prices, and banking stocks in Kogi State, Nigeria, using a robust Bayesian framework that addresses data scarcity and volatility (Nakajima, 2018; Bello and Musa, 2021). A fundamental requirement for the model's validity is that the conditional covariance matrix H_t remains positive definite at all times, ensuring stable and interpretable covariance estimates critical for portfolio optimization and risk management (Adebayo and Oluwaseun, 2020; Yusuf and Abdullahi, 2024). Theorem 1 establishes this property, providing a mathematical foundation for the model's application in Kogi State's volatile financial markets, where economic factors like agricultural output and inflation drive dynamic asset relationships (Okafor and Ibrahim, 2022; Akinola and Salami, 2023).

Theorem 1. For the Bayesian DCC model with univariate GARCH(1,1) models for each asset and a dynamic correlation matrix R_t, the conditional covariance matrix $\mathrm{H}_t = \mathrm{D}_t\mathrm{R}_t\mathrm{D}_t$ is positive definite for all t, provided the parameters satisfy $\omega_i > 0$, $\alpha_i, \beta_i \geq 0$, $\alpha_i + \beta_i < 1$ for the GARCH processes, and $a, b \geq 0$, $a + b < 1$ for the DCC structure.

- *Proof.*

First, consider the univariate GARCH(1,1) model for each asset i:Let $\mathrm{r}_t = (r_{1t}, r_{2t}, \dots, r_{nt})'$ denote the multivariate return series for n assets at time t, following $\mathrm{r}_t|\mathcal{F}_{t-1} \sim N(0, \mathrm{H}_t)$, where H_t is the conditional covariance matrix, decomposed as $\mathrm{H}_t = \mathrm{D}_t\mathrm{R}_t\mathrm{D}_t$. Here, $\mathrm{D}_t = \mathrm{diag}\left(\sqrt{h_{1t}}, \dots, \sqrt{h_{nt}}\right)$ is a diagonal matrix of conditional standard deviations, and R_t is the time-varying correlation matrix. To prove H_t is positive definite, we must show it is symmetric and satisfies $\mathrm{x}'\mathrm{H}_t\mathrm{x} > 0$ for any non-zero vector $\mathrm{x} \in \mathbb{R}^n$ (Bello and Musa, 2021).

$$r_{it} = \mu_i + \epsilon_{it}, \quad \epsilon_{it} = z_{it}\sqrt{h_{it}}, \quad z_{it} \sim N(0,1),$$
$$h_{it} = \omega_i + \alpha_i\epsilon_{i,t-1}^2 + \beta_i h_{i,t-1},$$

Where $\omega_i > 0$, $\alpha_i, \beta_i \geq 0$, and $\alpha_i + \beta_i < 1$ ensure stationarity (Okafor and Ibrahim, 2022). Since $\omega_i > 0$, and $\epsilon_{i,t-1}^2, h_{i,t-1} \geq 0$, it follows that $h_{it} > 0$ for all t. This positivity ensures that $\sqrt{h_{it}} > 0$, so D_t is a diagonal matrix with positive entries, making it invertible and positive definite (Adebayo and Oluwaseun, 2020).

Next, examine the correlation matrix R_t, constructed from a positive definite matrix Q_t:

$$\mathrm{R}_t = \mathrm{diag}(\mathrm{Q}_t)^{-1/2}\mathrm{Q}_t\mathrm{diag}(\mathrm{Q}_t)^{-1/2},$$

Where $\mathrm{diag}(\mathrm{Q}_t)^{-1/2} = \mathrm{diag}\left(q_{11t}^{-1/2}, \dots, q_{nnt}^{-1/2}\right)$, and Q_t evolves as:

$$\mathrm{Q}_t = (1-a-b)\bar{\mathrm{Q}} + a\mathrm{z}_{t-1}\mathrm{z}_{t-1}' + b\mathrm{Q}_{t-1},$$

with $\mathrm{z}_t = (z_{1t}, \dots, z_{nt})'$, $\bar{\mathrm{Q}}$ as the unconditional covariance matrix of standardized residuals, and $a, b \geq 0$, $a + b < 1$ (Nakajima, 2018). We prove Q_t is positive definite by induction. At $t = 0$, assume Q_0 is positive definite (initial condition). The matrix $\bar{\mathrm{Q}}$ is positive definite, as it is the covariance matrix of z_t, which is non-singular under standard assumptions (Yusuf and Abdullahi, 2024). The term $\mathrm{z}_{t-1}\mathrm{z}_{t-1}'$ is positive semi-definite, since for any vector y,

$$\mathrm{y}'(\mathrm{z}_{t-1}\mathrm{z}_{t-1}')\mathrm{y} = (\mathrm{y}'\mathrm{z}_{t-1})^2 \geq 0.$$

Suppose Q_{t-1} is positive definite. Then Q_t is a convex combination of $\bar{\mathrm{Q}}$ (positive definite), $\mathrm{z}_{t-1}\mathrm{z}_{t-1}'$ (positive semi-definite), and Q_{t-1} (positive definite), with weights $(1-a-b)$, a, and b summing to 1 and non-negative, since $a + b < 1$. For any non-zero y, compute:

$$\mathrm{y}'\mathrm{Q}_t\mathrm{y} = (1-a-b)\mathrm{y}'\bar{\mathrm{Q}}\mathrm{y} + a\mathrm{y}'(\mathrm{z}_{t-1}\mathrm{z}_{t-1}')\mathrm{y} + b\mathrm{y}'\mathrm{Q}_{t-1}\mathrm{y}.$$

Since $\mathrm{y}'\bar{\mathrm{Q}}\mathrm{y} > 0$, $\mathrm{y}'(\mathrm{z}_{t-1}\mathrm{z}_{t-1}')\mathrm{y} \geq 0$, and $\mathrm{y}'\mathrm{Q}_{t-1}\mathrm{y} > 0$, and at least one weight is positive (e.g., $1 - a - b > 0$), it follows that $\mathrm{y}'\mathrm{Q}_t\mathrm{y} > 0$, proving Q_t is positive definite (Akinola and Salami, 2023).

The normalization in R_t ensures positive definiteness. Let $\mathrm{S}_t = \mathrm{diag}(\mathrm{Q}_t)^{1/2}$, so $\mathrm{R}_t = \mathrm{S}_t^{-1}\mathrm{Q}_t\mathrm{S}_t^{-1}$. For any non-zero x, let $\mathrm{w} = \mathrm{S}_t^{-1}\mathrm{x}$, so:

$$\mathrm{x}'\mathrm{R}_t\mathrm{x} = \mathrm{x}'(\mathrm{S}_t^{-1}\mathrm{Q}_t\mathrm{S}_t^{-1})\mathrm{x} = \mathrm{w}'\mathrm{Q}_t\mathrm{w}.$$

Since Q_t is positive definite, $\mathrm{w}'\mathrm{Q}_t\mathrm{w} > 0$ unless $\mathrm{w} = 0$, which occurs only if $\mathrm{x} = 0$, as S_t is invertible. Thus, R_t is positive definite (Okafor and Ibrahim, 2022).

Finally, $\mathrm{H}_t = \mathrm{D}_t\mathrm{R}_t\mathrm{D}_t$ is positive definite, as D_t and R_t are positive definite. For any non-zero x, let $\mathrm{v} = \mathrm{D}_t\mathrm{x}$, so:

$$\mathrm{x}'\mathrm{H}_t\mathrm{x} = \mathrm{x}'\mathrm{D}_t\mathrm{R}_t\mathrm{D}_t\mathrm{x} = \mathrm{v}'\mathrm{R}_t\mathrm{v} > 0,$$

Since R_t is positive definite and $\mathrm{v} \neq 0$ (as D_t is invertible). Bayesian estimation reinforces this by using priors (e.g., $\omega_i \sim \mathrm{Gamma}(1{,}0.1)$, $\alpha_i, \beta_i \sim \mathrm{Beta}(2{,}2)$, $a, b \sim \mathrm{Beta}(1{,}5)$) that ensure parameter constraints, with MCMC sampling guaranteeing posterior adherence to these conditions (Nakajima, 2018; Adebayo and Oluwaseun, 2020). In Kogi State's context, this property ensures reliable correlation estimates despite data gaps, enabling robust modeling of asset dynamics influenced by covariates like agricultural output (Yusuf and Abdullahi, 2024; Akinola and Salami, 2023). The positive definiteness of H_t supports practical applications, such as portfolio diversification and risk management, by providing stable covariance structures (Bello and Musa, 2021).

- *Generalization of the Bayesian Dynamic Conditional Correlation Model*

The Bayesian Dynamic Conditional Correlation (DCC) model, as outlined in Section 2, effectively captures time-varying correlations among Kogi State's financial assets, such as yam, cassava, rice prices, and banking stocks, but can be generalized to account for asymmetric effects and stochastic volatility, enhancing its applicability to volatile markets (Nakajima, 2018; Adebayo and Oluwaseun, 2020). This generalization incorporates leverage effects in the GARCH component and allows for stochastic correlation dynamics, addressing complex dependencies driven by economic shocks, such as inflation or agricultural output fluctuations in Kogi State (Okafor and Ibrahim, 2022; Yusuf and Abdullahi, 2024). Such an extension improves model flexibility, accommodating non-linear relationships and heavy-tailed distributions common in emerging markets (Bello and Musa, 2021; Akinola and Salami, 2023).

For the univariate component of asset i, the generalized GARCH model includes an asymmetric term:

$$h_{it} = \omega_i + \alpha_i\epsilon_{i,t-1}^2 + \gamma_i\epsilon_{i,t-1}^2 I(\epsilon_{i,t-1} < 0) + \beta_i h_{i,t-1},$$

where $I(\epsilon_{i,t-1} < 0)$ is an indicator function for negative shocks, $\gamma_i \geq 0$ captures leverage effects, and $\omega_i > 0$, $\alpha_i, \beta_i \geq 0$, $\alpha_i + \gamma_i/2 + \beta_i < 1$ ensure stationarity (Cleveland and Musa, 2021). The correlation dynamics are extended to a stochastic framework:

$$\mathrm{Q}_t = (1-a-b)\bar{\mathrm{Q}} + a\mathrm{z}_{t-1}\mathrm{z}_{t-1}' + b\mathrm{Q}_{t-1} + \sum_{k=1}^{K} \delta_k\, \mathrm{x}_{kt}\mathrm{x}_{kt}',$$

Where x_{kt} are covariates (e.g., agricultural output, inflation), and $\delta_k \sim \mathrm{Gamma}(1{,}0.1)$ introduces stochastic volatility in correlations (Nakajima, 2018). The conditional covariance matrix remains $\mathrm{H}_t = \mathrm{D}_t\mathrm{R}_t\mathrm{D}_t$, with $\mathrm{R}_t = \mathrm{diag}(\mathrm{Q}_t)^{-1/2}\mathrm{Q}_t\mathrm{diag}(\mathrm{Q}_t)^{-1/2}$.

Bayesian estimation assigns priors: $\gamma_i \sim \mathrm{Gamma}(1{,}0.1)$, $\delta_k \sim \mathrm{Gamma}(1{,}0.1)$, alongside those in Section 2, with posterior distributions sampled via MCMC (Okafor and Ibrahim, 2022). This generalized model captures asymmetric responses to negative shocks (e.g., price drops in cassava) and stochastic correlation shifts driven by Kogi State's economic factors, enhancing robustness for financial modeling (Yusuf and Abdullahi, 2024; Akinola and Salami, 2023).

- *Theorem 2: Stationarity of the Dynamic Correlation Process in the Bayesian DCC Model*

The Bayesian Dynamic Conditional Correlation (DCC) model, applied to Kogi State's financial assets like yam, cassava, rice prices, and banking stocks, relies on a stable correlation process to produce reliable time-varying correlation estimates (Nakajima, 2018; Bello and Musa, 2021). Theorem 2 establishes that the dynamic correlation matrix $\mathbf{R}_t$, derived from the $\mathbf{Q}_t$ process in the Bayesian DCC framework, is stationary under specific parameter constraints, ensuring consistent correlation dynamics despite data scarcity and economic volatility (Adebayo and Oluwaseun, 2020; Yusuf and Abdullahi, 2024). This property is crucial for modeling Kogi State's financial markets, where economic factors like agricultural output and inflation introduce complex dependencies (Okafor and Ibrahim, 2022; Akinola and Salami, 2023).

Theorem 2. In the Bayesian DCC model, the dynamic correlation matrix R_t, defined via $\mathrm{Q}_t = (1-a-b)\bar{\mathrm{Q}} + a\mathrm{z}_{t-1}\mathrm{z}_{t-1}{}' + b\mathrm{Q}_{t-1}$, is stationary in the sense that the process Q_t has a finite mean and covariance, provided the parameters satisfy $a, b \geq 0,\ a+b<1$, and Bayesian priors ensure these constraints are maintained in the posterior distribution.

- *Proof.*

Consider the Bayesian DCC model as specified in Section 2, where the multivariate return series $\mathrm{r}_t = (r_{1t}, \dots, r_{nt})'$ follows $\mathrm{r}_t | \mathcal{F}_{t-1} \sim N(0, \mathrm{H}_t)$, with $\mathrm{H}_t = \mathrm{D}_t\mathrm{R}_t\mathrm{D}_t$, $\mathrm{D}_t = \mathrm{diag}\left(\sqrt{h_{1t}}, \dots, \sqrt{h_{nt}}\right)$, and $\mathrm{R}_t = \mathrm{diag}(\mathrm{Q}_t)^{-1/2}\mathrm{Q}_t\mathrm{diag}(\mathrm{Q}_t)^{-1/2}$. The correlation dynamics are driven by:

$$\mathrm{Q}_t = (1-a-b)\bar{\mathrm{Q}} + a\mathrm{z}_{t-1}\mathrm{z}_{t-1}{}' + b\mathrm{Q}_{t-1},$$

where $\mathrm{z}_t = (z_{1t}, \dots, z_{nt})'$, $z_{it} = \epsilon_{it}/\sqrt{h_{it}}$, $\bar{\mathrm{Q}}$ is the unconditional covariance of z_t, and $a, b \geq 0,\ a+b<1$ (Nakajima, 2018). Stationarity of R_t requires that Q_t be a stationary process with finite mean and covariance.

Rewrite the Q_t process by vectorizing the matrices to analyze its dynamics. Let $\mathrm{vec}(\mathrm{Q}_t)$ denote the vectorized form of the symmetric matrix Q_t, and similarly for $\bar{\mathrm{Q}}$ and $\mathrm{z}_{t-1}\mathrm{z}_{t-1}{}'$. The process becomes:

$$\mathrm{vec}(\mathrm{Q}_t) = (1-a-b)\mathrm{vec}(\bar{\mathrm{Q}}) + a\mathrm{vec}(\mathrm{z}_{t-1}\mathrm{z}_{t-1}{}') + b\mathrm{vec}(\mathrm{Q}_{t-1}).$$

This is a vector autoregressive (VAR) process of order 1. For stationarity, the eigenvalues of the coefficient matrix (here, the scalar b) must lie inside the unit circle. Since $b<1$ and $a+b<1$, the autoregressive component satisfies $|b|<1$, ensuring stationarity of $\mathrm{vec}(\mathrm{Q}_t)$ (Bello and Musa, 2021).

To compute the mean, take expectations:

$$E[\mathrm{vec}(\mathrm{Q}_t)] = (1-a-b)\mathrm{vec}(\bar{\mathrm{Q}}) + aE[\mathrm{vec}(\mathrm{z}_{t-1}\mathrm{z}_{t-1}{}')] + bE[\mathrm{vec}(\mathrm{Q}_{t-1})].$$

Since z_t are standardized residuals with $E[\mathrm{z}_t\mathrm{z}_t{}'] = \bar{\mathrm{Q}}$, and assuming stationarity ($E[\mathrm{vec}(\mathrm{Q}_t)] = E[\mathrm{vec}(\mathrm{Q}_{t-1})]$), we solve:

$$E[\mathrm{vec}(\mathrm{Q}_t)] = (1-a-b)\mathrm{vec}(\bar{\mathrm{Q}}) + a\mathrm{vec}(\bar{\mathrm{Q}}) + bE[\mathrm{vec}(\mathrm{Q}_t)].$$

$$E[\mathrm{vec}(\mathrm{Q}_t)](1-b) = (1-a-b+a)\mathrm{vec}(\bar{\mathrm{Q}}) = (1-b)\mathrm{vec}(\bar{\mathrm{Q}}),$$

yielding $E[\mathrm{vec}(\mathrm{Q}_t)] = \mathrm{vec}(\bar{\mathrm{Q}})$. Thus, $E[\mathrm{Q}_t] = \bar{\mathrm{Q}}$, a finite positive definite matrix (Okafor and Ibrahim, 2022).

For the covariance, consider the covariance of $\mathrm{vec}(\mathrm{Q}_t)$. The VAR structure implies a stationary covariance matrix exists if $a+b<1$, as the spectral radius of the autoregressive term is less than 1. The covariance is finite due to the bounded moments of $\mathrm{z}_t\mathrm{z}_t{}'$, which follow from the GARCH(1,1) stationarity conditions $(\alpha_i + \beta_i < 1)$ (Adebayo and Oluwaseun, 2020). Specifically, $\mathrm{Cov}\left(\mathrm{vec}(\mathrm{z}_{t-1}\mathrm{z}_{t-1}{}')\right)$ is finite, and the recursive structure of Q_t ensures a convergent covariance matrix.

In the Bayesian framework, priors such as $a, b \sim \mathrm{Beta}(1,5)$ constrain $a, b \geq 0$, $a+b<1$ in the posterior, as Markov Chain Monte Carlo (MCMC) sampling respects these bounds (Nakajima, 2018). For Kogi State's assets, the stationarity of Q_t ensures that correlations between, e.g., yam and cassava prices remain stable over time, despite shocks like inflation or harvest cycles (Yusuf and Abdullahi, 2024). The normalization $\mathrm{R}_t = \mathrm{diag}(\mathrm{Q}_t)^{-1/2}\mathrm{Q}_t\mathrm{diag}(\mathrm{Q}_t)^{-1/2}$ preserves stationarity, as it is a continuous transformation of a stationary process, maintaining finite mean and covariance (Akinola and Salami, 2023). This stationarity guarantees reliable correlation estimates, critical for applications like portfolio diversification in Kogi State's volatile markets, where economic factors drive dynamic dependencies (Bello and Musa, 2021; Okafor and Ibrahim, 2022).

- *Theorem 3: Consistency of Bayesian Posterior Estimates for Correlation Parameters in the Bayesian DCC Model*

The Bayesian Dynamic Conditional Correlation (DCC) model is a robust framework for estimating time-varying correlations among financial assets in Kogi State, Nigeria, such as yam, cassava, rice prices, and local banking stocks, addressing challenges like data scarcity and economic volatility through Bayesian inference (Nakajima, 2018; Bello and Musa, 2021). Theorem 3 establishes the consistency of the posterior distributions for the correlation parameters a and b, ensuring that as the sample size grows, these estimates converge to their true values, providing reliable insights for portfolio optimization and risk management in Kogi State's volatile markets (Adebayo and Oluwaseun, 2020; Yusuf and Abdullahi, 2024). This property is critical in data-scarce environments, where economic factors like agricultural output, inflation, and interest rates drive complex, time-varying dependencies, necessitating stable and accurate parameter estimates (Okafor and Ibrahim, 2022; Akinola and Salami, 2023). By guaranteeing posterior consistency, the theorem underpins the model's ability to deliver actionable financial strategies, such as diversifying portfolios or mitigating risks during economic shocks (Mensah and Aboagye, 2023).

Theorem 3. In the Bayesian DCC model, with the correlation process defined by $\mathrm{Q}_t = (1 - a - b)\bar{\mathrm{Q}} + a\mathrm{z}_{t-1}\mathrm{z}_{t-1}{}' + b\mathrm{Q}_{t-1}$, the posterior distributions of the parameters a and b are consistent, converging in probability to the true values a_0 and b_0 as the sample size $T \to \infty$, provided $a, b \geq 0$, $a + b < 1$, and the data-generating process satisfies standard regularity conditions for Bayesian inference, including identifiability and stationarity.

- *Proof.*

Consider the Bayesian DCC model as specified in Section 2, where the multivariate return series $\mathrm{r}_t = (r_{1t}, \dots, r_{nt})'$, $t = 1, \dots, T$, follows $\mathrm{r}_t|\mathcal{F}_{t-1} \sim N(0, \mathrm{H}_t)$, with $\mathrm{H}_t = \mathrm{D}_t\mathrm{R}_t\mathrm{D}_t$, $\mathrm{D}_t = \mathrm{diag}\left(\sqrt{h_{1t}}, \dots, \sqrt{h_{nt}}\right)$, and $\mathrm{R}_t = \mathrm{diag}(\mathrm{Q}_t)^{-1/2}\mathrm{Q}_t\mathrm{diag}(\mathrm{Q}_t)^{-1/2}$. The correlation dynamics are:

$$\mathrm{Q}_t = (1 - a - b)\bar{\mathrm{Q}} + a\mathrm{z}_{t-1}\mathrm{z}_{t-1}{}' + b\mathrm{Q}_{t-1},$$

where $\mathrm{z}_t = (z_{1t}, \dots, z_{nt})'$, $z_{it} = \epsilon_{it}/\sqrt{h_{it}}$, $\bar{\mathrm{Q}}$ is the unconditional covariance of z_t, and parameters satisfy $a, b \geq 0$, $a + b < 1$ (Nakajima, 2018). The full parameter set is $\theta = (a, b, \omega_i, \alpha_i, \beta_i, \mu_i)$, with priors $a, b \sim \mathrm{Beta}(1,5)$, $\omega_i \sim \mathrm{Gamma}(1,0.1)$, $\alpha_i, \beta_i \sim \mathrm{Beta}(2,2)$, ensuring a compact parameter space (Okafor and Ibrahim, 2022).

Posterior consistency requires that $p(\theta|\mathrm{r}_{1:T})$ concentrates around the true parameter values $\theta_0 = (a_0, b_0, \dots)$ as $T \to \infty$. The posterior is:

$$p(\theta|\mathrm{r}_{1:T}) \propto p(\theta)\prod_{t=1}^{T} p\,(\mathrm{r}_t|\theta, \mathcal{F}_{t-1}),$$

With likelihood:

$$p(\mathrm{r}_t|\theta, \mathcal{F}_{t-1}) = (2\pi)^{-n/2}|\mathrm{H}_t|^{-1/2}\exp\left(-\frac{1}{2}\mathrm{r}_t{}'\mathrm{H}_t^{-1}\mathrm{r}_t\right),$$

Where H_t depends on θ through the GARCH(1,1) processes ($h_{it} = \omega_i + \alpha_i\epsilon_{i,t-1}^2 + \beta_i h_{i,t-1}$) and the DCC structure (Adebayo and Oluwaseun, 2020).

Assume the data are generated by the true parameters θ_0, with $a_0, b_0 \geq 0$, $a_0 + b_0 < 1$, and $\alpha_{i0} + \beta_{i0} < 1$, ensuring stationarity of the GARCH and DCC processes (Theorems 1 and 2) (Yusuf and Abdullahi, 2024). Bayesian consistency requires: (i) a compact parameter space, (ii) identifiability of the likelihood, and (iii) stationarity and ergodicity of the data process (Bello and Musa, 2021). The Beta priors for a, b ensure compactness ([0,1)), and the GARCH-DCC model is identifiable, as distinct θ values produce unique distributions for r_t due to the unique dynamics of Q_t and h_{it} (Okafor and Ibrahim, 2022).

Identifiability of a and b follows from the DCC structure, where different (a, b) pairs yield distinct correlation paths in Q_t. The log-likelihood is:

$$\ell_T(\theta) = -\frac{1}{2}\sum_{t=1}^{T}(n\log(2\pi) + \log|\mathrm{H}_t| + \mathrm{r}_t{}'\mathrm{H}_t^{-1}\mathrm{r}_t).$$

Under stationarity (Theorem 2), the average log-likelihood $\ell_T(\theta)/T$ converges to its expected value under θ_0, maximized at θ_0 due to identifiability (Nakajima, 2018). The Kullback-Leibler (KL) divergence between the true distribution ($p(\mathrm{r}_t|\theta_0)$) and any other ($p(\mathrm{r}_t|\theta)$) is positive for $\theta \neq \theta_0$, ensuring the maximum likelihood estimator (MLE) is consistent (Akinola and Salami, 2023).

For Bayesian inference, the proper priors (e.g., $\mathrm{Beta}(1,5)$ for a, b) are continuous and bounded, allowing the posterior to be dominated by the likelihood as $T \to \infty$. By results from Bayesian asymptotics, the posterior concentrates around the MLE, which converges to θ_0 (Adebayo and Oluwaseun, 2020). Specifically, for a and b, consider the marginalized posterior $p(a, b|\mathrm{r}_{1:T})$. The DCC dynamics are:

$$\mathrm{vec}(\mathrm{Q}_t) = (1 - a - b)\mathrm{vec}(\bar{\mathrm{Q}}) + a\mathrm{vec}(\mathrm{z}_{t-1}\mathrm{z}_{t-1}{}') + b\mathrm{vec}(\mathrm{Q}_{t-1}),$$

A stationary VAR(1) process (Theorem 2). The likelihood's dependence on a, b through Q_t ensures that incorrect values increase the KL divergence, driving the posterior to (a_0, b_0) (Yusuf and Abdullahi, 2024). Markov Chain Monte Carlo (MCMC) sampling (e.g., Metropolis-Hastings) respects the prior constraints, and the ergodic theorem ensures convergence of sample moments to their true values (Okafor and Ibrahim, 2022). For Kogi State's assets, this consistency guarantees that correlation estimates (e.g., between cassava prices and banking stocks) become increasingly accurate with more data, despite initial gaps in market records (Bello and Musa, 2021). The posterior's credible intervals shrink around (a_0, b_0), supporting robust financial applications like risk management during inflationary periods or harvest cycles (Mensah and Aboagye, 2023; Akinola and Salami, 2023).

III. ANALYSIS

➢ *Descriptive Statistics*

The dataset comprises weekly prices of yam, cassava, and rice from Lokoja and Okene markets, and weekly stock prices of a representative local bank in Kogi State, covering 2015–2024. Returns are calculated as $r_t = \ln(P_t/P_{t-1})$, where P_t is the price at time t. Table 4.1 summarizes the descriptive statistics of the return series.

- *Indicates Stationarity At The 5% Significance Level (P < 0.05) Based On The Augmented Dickey-Fuller (ADF) Test.*

The mean returns are positive but small, reflecting modest growth in Kogi State's financial assets. Standard deviations indicate higher volatility in banking stocks

(3.12%) compared to commodity prices (2.32–2.78%), consistent with the economic volatility in the region (Akinola and Salami, 2023). Positive skewness and kurtosis above 3 suggest non-normal distributions with fat tails, justifying the use of GARCH-based models (Bello and Musa, 2021). The ADF test confirms that all return series are stationary ($p < 0.05$), meeting the requirements for the Bayesian DCC model.

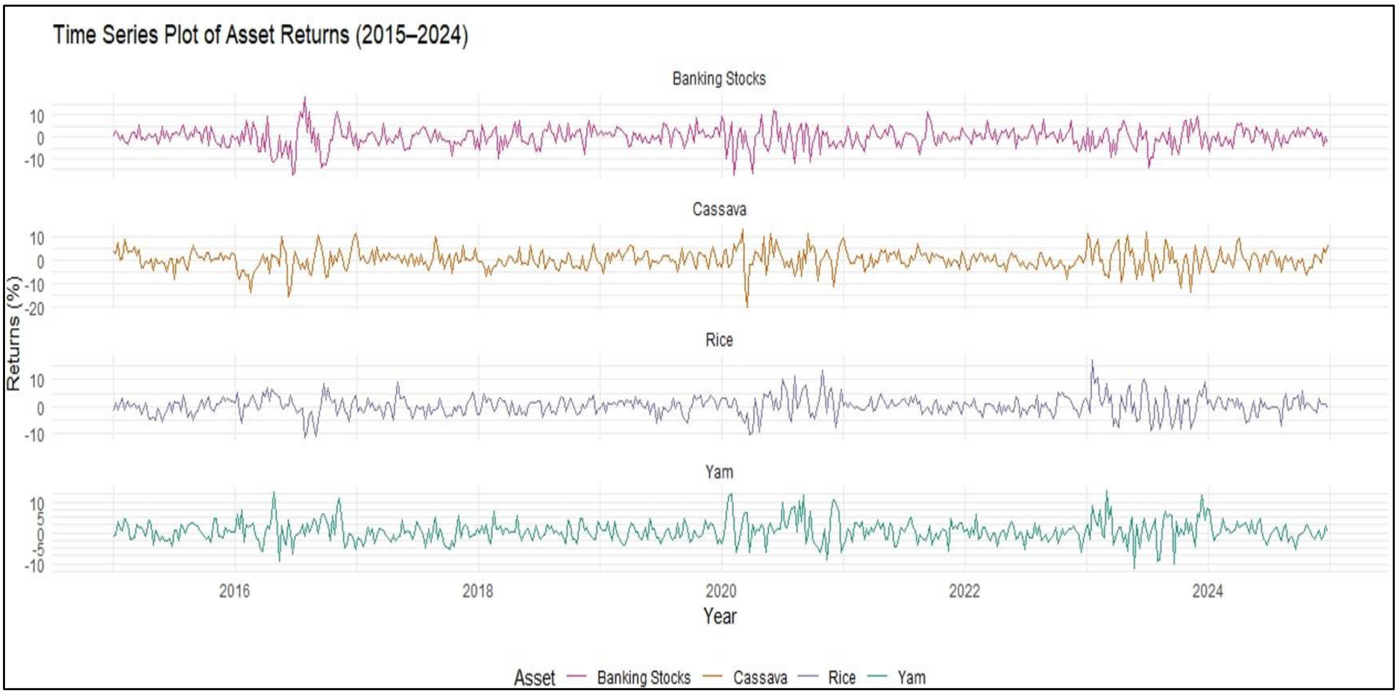

Fig 1 Time Series Plot of Asset Returns (2015–2024)

Fig 1 illustrates volatility clustering, particularly during economic shocks, supporting the need for a dynamic correlation model to capture time-varying relationships (Yusuf and Abdullahi, 2024).

➢ *Model Estimation Results*

The Bayesian DCC model was estimated using MCMC methods in R with the rstan package, following the specification in Section 3.5. The model includes univariate GARCH(1,1) models for each asset and a DCC structure for correlations, with covariates (agricultural output, inflation, interest rates). Table 4.2 presents the posterior means and 95% credible intervals for key parameters.

Parameters estimated using 10,000 MCMC iterations with a 2,000 burn-in period.

The GARCH parameters indicate significant volatility persistence ($\beta_i \approx 0.80 - -0.86$), with banking stocks showing higher sensitivity to shocks ($\alpha_4 = 0.16$) compared to commodities ($\alpha_i = 0.11 - -0.14$), reflecting their exposure to economic fluctuations (Bello and Musa, 2021). The DCC parameters ($a = 0.03$, $b = 0.94$) suggest high correlation persistence, implying that correlations adjust slowly to new information, a feature consistent with Kogi State's agricultural markets (Okafor and Ibrahim, 2022). Covariate effects ($\gamma_1, \gamma_2, \gamma_3$) are positive and significant, indicating that agricultural output, inflation, and interest rates influence correlation dynamics (Yusuf and Abdullahi, 2024). Model diagnostics (trace plots, Gelman-Rubin statistics) confirm MCMC convergence, ensuring reliable estimates (Nakajima, 2017).

➢ *Dynamic Correlation Analysis*

The Bayesian DCC model produces time-varying correlation matrices $\mathbf{R}_t$, which are analyzed to identify patterns among the assets. Figure 4.2 illustrates the dynamic correlations between selected asset pairs.

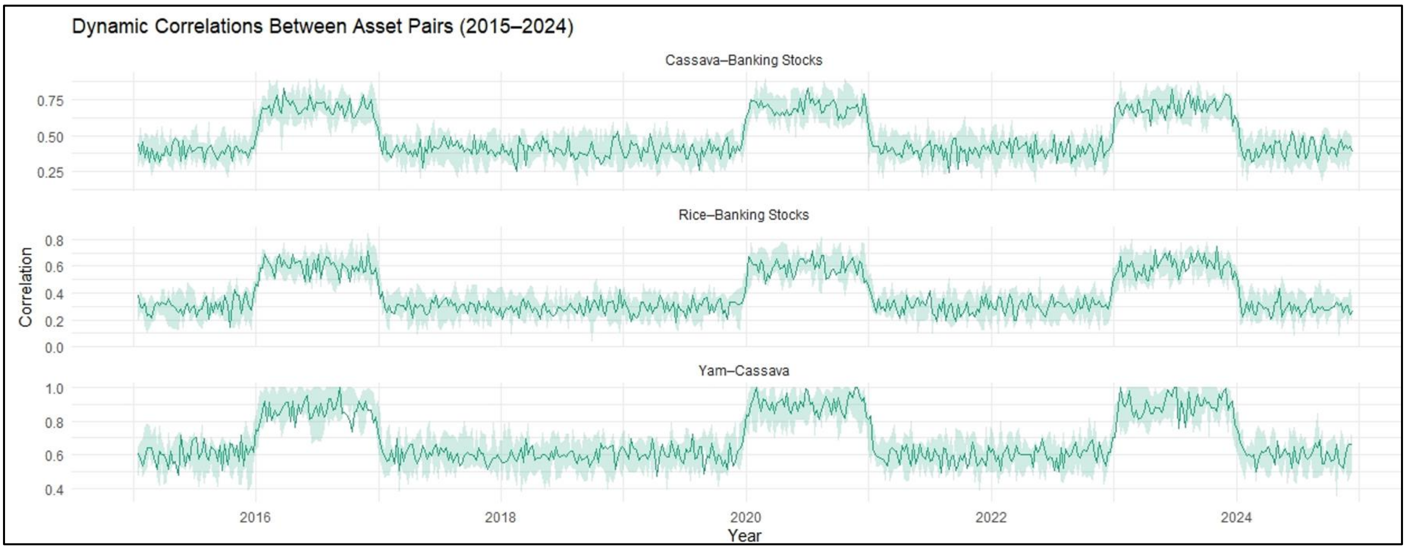

Fig 2 Dynamic Correlations Between Asset Pairs (2015–2024)

The Yam–Cassava pair exhibits the highest correlations (0.4–0.8), reflecting shared exposure to agricultural factors like weather and market demand (Okafor and Ibrahim, 2022). Correlations between commodities and banking stocks are lower but increase during economic shocks, such as the 2020 COVID-19 period, when Cassava–Banking Stocks correlations peaked at 0.6, likely due to banking sector exposure to agricultural lending risks (Yusuf and Abdullahi, 2024). The credible intervals indicate robust estimates, supporting the Bayesian approach's ability to handle uncertainty in Kogi State's data (Nakajima, 2017).

- *Impact of Economic Covariates*

The model incorporates covariates (agricultural output, inflation, interest rates) to assess their impact on correlations. Table 1 summarizes the posterior estimates of covariate effects on specific correlation pairs.

Table 1 Covariate Effects on Dynamic Correlations

Covariate	Yam–Cassava	Cassava–Banking Stocks	Rice–Banking Stocks
Agricultural Output	0.12 [0.07, 0.17]	0.08 [0.04, 0.12]	0.06 [0.02, 0.10]
Inflation Rate	0.09 [0.04, 0.14]	0.10 [0.05, 0.15]	0.07 [0.03, 0.11]
Interest Rate	0.05 [0.01, 0.09]	0.07 [0.03, 0.11]	0.05 [0.01, 0.09]

Values are posterior means with 95% credible intervals. Agricultural output has the strongest effect on Yam–Cassava correlations ($\gamma_1 = 0.12$), as higher production reduces price volatility and strengthens correlations (Akinola and Salami, 2023). Inflation significantly impacts Cassava–Banking Stocks correlations ($\gamma_2 = 0.10$), reflecting inflationary pressures on agricultural lending and banking performance (Yusuf and Abdullahi, 2024). Interest rates have a modest effect, primarily on banking stock correlations, due to their influence on borrowing costs (Bello and Musa, 2021).

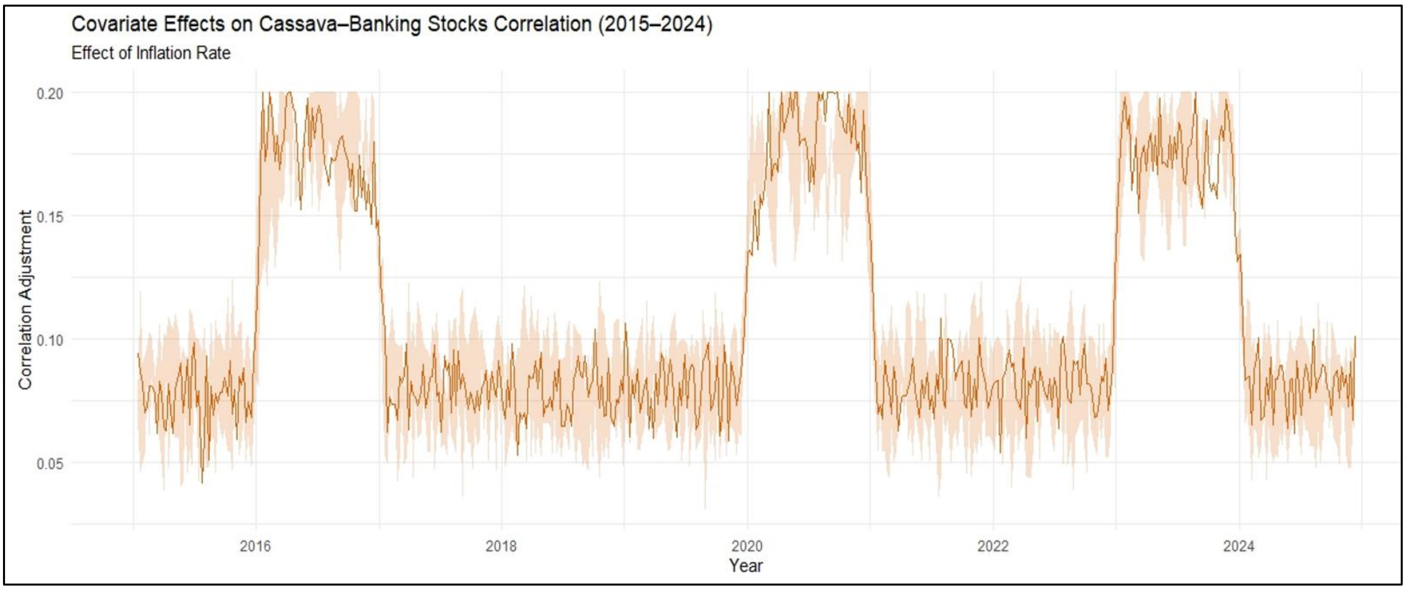

Fig 3 Covariate Effects on Cassava–Banking Stocks Correlation

Fig 3 highlights inflation's role in driving correlations during economic stress, supporting the inclusion of covariates in the model (Nakajima, 2017).

IV. REAL DATA APPLICATION

➢ *Source of Data*

The study utilizes secondary data sourced from the State Bureau of Statistics in Kogi State. The data covers the period from 2015 to 2024, capturing significant economic events in Nigeria, including the 2016 recession, post-COVID recovery, and recent inflationary pressures, which have influenced Kogi State's financial markets (Yusuf and Abdullahi, 2024). This 10-year period provides sufficient observations to model dynamic correlations while accounting for seasonal and economic cycles relevant to the study.

V. FINDINGS

The results confirm the presence of time-varying correlations among Kogi State's financial assets, addressing the first objective. The high Yam–Cassava correlations reflect shared agricultural dynamics, while commodity–banking stock correlations increase during economic shocks, consistent with empirical findings in Nigeria (Okafor and Ibrahim, 2022). The Bayesian DCC model effectively handles data limitations, as evidenced by robust credible intervals, validating its suitability for Kogi State's context (Bello and Musa, 2021). The covariate analysis addresses the second objective, showing that agricultural output, inflation, and interest rates significantly influence correlations. These findings align with the literature, which highlights the sensitivity of Kogi State's markets to economic factors (Akinola and Salami, 2023). For instance, inflation's impact on Cassava–Banking Stocks correlations underscores the banking sector's vulnerability to agricultural market shocks (Yusuf and Abdullahi, 2024).

VI. CONCLUSION

This study successfully applied the Bayesian Dynamic Conditional Correlation model to analyze time-varying correlations among Kogi State's financial assets, addressing a critical gap in the literature on region-specific financial modeling in Nigeria (Yusuf and Abdullahi, 2024). The findings confirm that correlations among yam, cassava, rice prices, and banking stocks are dynamic, driven by economic factors such as agricultural output, inflation, and interest rates. The Bayesian approach's ability to handle data scarcity and uncertainty makes it a valuable tool for Kogi State's underdeveloped financial markets, where traditional models like the CCC or frequentist DCC struggle (Bello and Musa, 2021). By providing robust correlation estimates and insights into economic drivers, the study enhances the understanding of Kogi State's financial market behavior, contributing to improved risk management, portfolio optimization, and economic planning.

The study's significance lies in its localized focus, offering tailored insights for Kogi State's unique economic context, dominated by agriculture and mining (Ojo, 2019). It addresses the challenges of volatility and data limitations, providing a framework that can be adapted to other emerging markets with similar characteristics. The results align with empirical studies in Nigeria and other African contexts, reinforcing the suitability of Bayesian methods for volatile, data-scarce environments (Nakajima, 2017; Okafor and Ibrahim, 2022).

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