Development of a Mixed Method for Estimation of Generalized Extreme Value Distribution with Application in Hydrology

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Abstract

The Generalized Extreme Value (GEV) distribution is widely used for modelling extreme environmental events such as rainfall, floods, and wind speeds. This study proposes a Mixed Estimation Method (MEM) that combines Trimmed L-Moments (1,1) and Maximum Likelihood Estimation for improved parameter estimation of the GEV distribution. The performance of the proposed method was evaluated using Monte Carlo simulation for different sample sizes and compared with MLE, L-Moments, and TL-Moments (1,1) using Mean Squared Error (MSE) as the criterion. Results showed that the MEM consistently produced more accurate parameter estimates with lower MSE values. The method was also applied to annual maximum rainfall data from Ogun State, Nigeria, where it outperformed the other estimation methods. Return level analysis indicated increasing rainfall magnitudes with higher return periods, suggesting potential future extreme rainfall events. The study concludes that the proposed MEM is a robust and efficient approach for GEV parameter estimation in extreme value and hydrological studies.

Keywords: *Generalized Extreme Value Distribution, Maximum Likelihood Estimation, Trimmed L-Moments, Mixed Estimation Method, Monte Carlo Simulation.*

I. INTRODUCTION

Long-term changes in the statistical distribution of weather patterns can be thought of as climate change. These changes have made the seasons unpredictable, posing one of the greatest threats to humans. Recent assessments show that climate variability has intensified, leading to more frequent and severe extreme rainfall events, with significant implications for ecosystems and human systems (IPCC, 2021; IPCC, 2022). Among the indicators of climate change, extreme rainfall is particularly critical due to its direct association with floods, landslides, and other hydrological disasters that pose serious risks to life, property, and infrastructure (WMO, 2023).

In Nigeria, the impact of extreme rainfall has been especially severe, with recurring flood events causing widespread displacement, destruction of property, and loss of livelihoods. The devastating flood events of 2012 and 2013 remain notable examples, affecting thousands of households and resulting in extensive damage to homes, agricultural lands, transportation networks, and public facilities (NEMA, 2013). Similar flood incidents continue to be reported in major urban and coastal centres such as Lagos, Port Harcourt, and Calabar, demonstrating the country's growing vulnerability to hydrological extremes and climate-related hazards.

Globally, flood disasters have increased in both frequency and intensity, affecting millions of people annually. Recent disaster statistics indicate that flooding accounts for a substantial proportion of weather-related disasters worldwide, causing significant economic losses, environmental degradation, and human displacement (WMO, 2023; EM-DAT, 2024). Furthermore, studies have shown that extreme hydrological events contribute substantially to agricultural losses, infrastructure failure, food insecurity, and public health crises, including outbreaks of waterborne diseases and malnutrition (IPCC, 2022). These developments emphasize the urgent need for reliable statistical tools capable of accurately characterizing extreme rainfall behaviour.

The statistical modelling of extreme rainfall has therefore become a major area of research in hydrology, climatology, and environmental sciences. Extreme Value Theory (EVT) provides an appropriate framework for analyzing rare and extreme events through distributions specifically designed to model maxima or minima of observations. Among these distributions, the Generalized Extreme Value (GEV) distribution occupies a central position because it unifies the Gumbel, Fréchet, and Weibull families within a single modelling framework. The flexibility of the GEV distribution enables it to represent different tail behaviours commonly observed in environmental and climatic datasets.

Several studies have investigated the suitability of extreme value distributions for modelling environmental data in Nigeria and beyond. In a notable study, Ilesanmi *et al.,* (2024) applied the Generalized Extreme Value distribution to annual maximum rainfall data recorded in Akure, Ondo State, Nigeria, covering the period from 1981 to 2019. The authors employed the Maximum Likelihood Estimation (MLE) technique to estimate the model parameters and assessed the adequacy of the fitted model for describing extreme rainfall events. Their findings revealed that the estimated shape parameter exhibited Fréchet-type behaviour, indicating the presence of a heavy-tailed distribution suitable for modelling exceptionally large rainfall amounts. The study further demonstrated the applicability of extreme value models in hydrological risk assessment and emphasized the importance of selecting appropriate distributions for predicting extreme rainfall occurrences.

Beyond identifying suitable extreme value models, attention has increasingly shifted toward the efficiency of parameter estimation procedures. Accurate estimation of the location, scale, and shape parameters of the GEV distribution is critical because these parameters directly influence return level estimation, risk assessment, and forecasting of future extreme events. Consequently, several estimation methods have been proposed, including Maximum Likelihood Estimation (MLE), Probability Weighted Moments (PWM), L-moments, Trimmed L-moments (TL-moments), and Bayesian estimation techniques.

In another important contribution, Ilesanmi *et al.,* (2024) conducted a comprehensive performance evaluation of selected estimators for the Generalized Extreme Value distribution using annual maximum rainfall data obtained from the Nigerian Meteorological Agency (NiMet) between 1980 and 2021. The results revealed significant differences in the performance of the estimation methods. Specifically, the TL-moments (1,1) estimator consistently produced the lowest MSE values for both Ekiti and Ondo States.

Despite the advances achieved through existing estimation methods, modelling extremes remains challenging because of the rarity, variability, and heavy-tailed nature of extreme rainfall observations. Traditional estimation techniques may exhibit bias, instability, or reduced efficiency, particularly when sample sizes are small or when datasets contain unusually large observations (Coles, 2001). This study therefore focuses on the development of a Mixed Estimation Method for the Generalized Extreme Value (GEV) distribution. The proposed method seeks to improve estimation accuracy and reliability in the modelling of extreme rainfall data by integrating the desirable properties of existing estimation techniques. The developed approach is subsequently applied to observed rainfall records for the estimation of return levels and return periods, thereby providing valuable information for flood risk assessment, disaster preparedness, water resource management, and climate adaptation planning in Nigeria.

II. GENERALIZED EXTREME VALUE (GEV) DISTRIBUTION

The generalized extreme-value (GEV) distribution has been used to model a wide range of natural extremities in a variety of applications, including floods, rainfall, wind speeds, wave height, and other situations. Its limiting maximum cumulative distribution function (CDF) is of the form.

$$F(x) = exp^{-\left[1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right]^{\frac{1}{\alpha}}} \quad (1)$$

Its probability density function (PDF) is given as

$$f(x) = \left[1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right]^{\frac{1}{\alpha}-1} exp^{-\left[1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right]^{\frac{1}{\alpha}}} \quad (2)$$

Where $\sigma > 0$ is the scale parameter, $-\infty < \alpha < \infty$ is the shape parameter and $\mu > 0$ is the location parameter. The value of α dictates the tail behaviour of CDF. When $\alpha = 0$ is the Gumbel distribution, when $\alpha < 0$ is the Fréchet and when $\alpha > 0$ is the Weibull distribution.

- *Data and Methodology*

In this work, the GEV distribution is fitted to rainfall data set of two selected state in south-western zone in Nigeria. The data was obtained from Nigeria metrological agency (NIMET) from 1980-2021 in Lagos. Three different estimation methods were employed to estimate the location, scale and shape parameters of generalized extreme value distribution They include, Maximum likelihood method, L-moments and TL- moments (1,1).

- *Maximum Likelihood Estimation*

If the set $\{x_i\}$ are independent and identically distributed for a GEV distribution, then the likelihood and the log-likelihood function for a sample of n observations $\{x_{1,} x_{2,} \ldots x_n\}$ are given as follows;

$$L(\mu, \sigma, \alpha) = f\left(x_{1,} x_{2,} \ldots, x_{n,} / \mu, \sigma, \alpha\right) = \prod_{i=1}^{n} f(x_i / \mu, \sigma, \alpha) \quad (3)$$

$$L = \left(\frac{1}{\sigma}\right)^n e^{-\sum_{i=1}^{n}\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{\frac{1}{\sigma}}} \prod_{i=1}^{n}\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{\frac{1}{\sigma}-1} \quad (4)$$

The log-likelihood function is given by

$$l = -nln(\sigma) - \left(1-\frac{1}{\sigma}\right)\sum_{i=1}^{n} ln\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right) - \sum_{i=1}^{n}\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{\frac{1}{\sigma}} \tag{5}$$

The ML method requires the computation of the first-order partial derivatives of the log-likelihood function with respect to each of its parameters, equating them to zero and then solving the resultant equations. This yields the system of equations presented as follows:

$$\frac{\partial l}{\partial \mu} = -\frac{\alpha}{\sigma}\left(1-\frac{1}{\sigma}\right)\sum_{i=1}^{n}\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{-1} - \left(\frac{1}{\alpha}\right)\sum_{i=1}^{n}\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{\frac{1}{\alpha}-1} = 0 \tag{6}$$

$$\frac{\partial l}{\partial \sigma} = -\frac{n}{\sigma} + \left(1-\frac{1}{\sigma}\right)\left(-\frac{\alpha}{\sigma^2}\right)\sum_{i=1}^{n}\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{-1}(x-\mu) + \left(\frac{1}{\alpha}\right)\sum_{i=1}^{n}\left(-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{-1}\left(\frac{\alpha}{\sigma^2}\right)(x-\mu)\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{\frac{1}{\alpha}} = 0 \tag{7}$$

$$\frac{\partial l}{\partial \alpha} = \frac{1}{\sigma}\sum_{i=1}^{n}\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{\frac{1}{\alpha}-1}\left(\frac{x-\mu}{\sigma}\right) + \left(1-\frac{1}{\alpha}\right)\sum_{i=1}^{n}\left(1\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{-1}\left(\frac{x-\mu}{\sigma}\right) - \left(\frac{1}{\alpha^2}\right)\sum_{i=1}^{n} ln\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right) +$$
$$\left(\frac{1}{\alpha^2}\right)\sum_{i=1}^{n} ln\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{\frac{1}{\alpha}} = 0 \tag{8}$$

As there is no analytical solution for obtaining the maximum likelihood estimates of the GEV parameters, then, the three systems of the log likelihood equations will be solved by numerical means to obtain the parameters μ, σ and α.

- *L-Moments Estimation Method*

Let X_1, X_2, X_3,…., X_r be a random sample of size r and let $X_{1:r}$, $X_{2:r}$, $X_{3:r}$, …$X_{r:r}$ denote the corresponding order statistics. The r[th] L-moment defined by (Hosking 1990) as:

$$\lambda_r = \frac{1}{r}\sum_{k=0}^{r-1}(-1)^k \binom{r-1}{k} E(X_{r-k:r}) \tag{9}$$

And

$$E(X_{r-k:r}) = \frac{r!}{(i-1)!\,(r-1)!}\int_0^1 x(F)\,F^{i-1}(1-F)^{r-i}dF \tag{10}$$

Therefore

$$\lambda_r = \frac{1}{r}\sum_{k=0}^{r-1}(-1)^k \binom{r-1}{k}\frac{r!}{(i-1)!\,(r-1)!}\int_0^1 x(F)\,F^{i-1}(1-F)^{r-i}dF \tag{11}$$

Where $x(F)$ and F are quantile function and cumulative distribution, respectively.

The first four (Population) L-moment ratio were then defined by putting r = 1,2,3 and 4 is

$$\lambda_1 = E(X_{1:1}) = \int_0^1 x(F)\,dF \tag{12}$$

$$\lambda_2 = \frac{1}{2}E(X_{2:2} - X_{1:2}) = \int_0^1 x(F)\,(2F-1)dF \tag{13}$$

$$\lambda_3 = \frac{1}{3}E(X_{3:3} - 2X_{2:3} + X_{1:3}) = \int_0^1 x(F)\,(6F^2 - 6F + 1)dF \tag{14}$$

$$\lambda_4 = \frac{1}{4}E(X_{4:4} - 3X_{3:4} - 3X_{2:4} + X_{1:4}) = \int_0^1 x(F)\,(20F^3 - 30F^3 + 12F^2 - 1)dF \tag{15}$$

The coefficients of the variation, skewness and kurtosis of the probability density function are

$$\tau = \frac{\lambda_2}{\lambda_1}, \qquad \tau_3 = \frac{\lambda_3}{\lambda_2} \qquad \tau_4 = \frac{\lambda_4}{\lambda_2}$$

The sample L-moments are estimated from the sample order statistics which are defined by

$$l_r = \frac{1}{(r)\binom{n}{r}}\sum_{i=1}^{n}\sum_{k=0}^{r-1}(-1)^k \binom{r-1}{k}\binom{i-1}{r-1-k}\binom{n-i}{k}x_{i:n} \tag{16}$$

The equations can be obtained by equating the first three population L-moments to the corresponding first three sample L-moments, i.e., $\lambda_1 = l_1$, $\lambda_2 = l_2$, and $\lambda_3 = l_3$.

Substituting the values of the population L-moments λ_1, λ_2, and λ_3 for the sample L-moments,

The L-moments estimator of the parameters of the GEV distribution are given by

$$\hat{\mu} = l_1 - \frac{\sigma}{\alpha}[1 - \Gamma(\alpha + 1)] \tag{17}$$

$$\hat{\sigma} = \frac{\alpha l_2}{\Gamma\alpha + 1(1 - 2^{-\alpha})} \tag{18}$$

$$\text{L-skewness} = \tau_3 = \frac{\lambda_3}{\lambda_2} = \frac{3(2^{-\alpha}) - 2(3^{-\alpha}) - 1}{1 - (2^{-\alpha})} \tag{19}$$

$$\text{L-kurtosis} = \tau_4 = \frac{\lambda_4}{\lambda_2} = \frac{10(3^{-\alpha}) - 5(3^{-\alpha}) + 6(2^{-\alpha}) + 1}{1 - (2^{-\alpha})} \tag{20}$$

The shape parameter of the GEV will be estimated by using some approximation discussed by Hosking and Wallis (2005) i.e.

$$\hat{\alpha} = 7.8590\hat{c} + 2.95554\hat{c}^2$$

Where

$$\hat{c} = \frac{2}{3 + \tau_3} - \frac{log2}{log3} \tag{21}$$

- *TL- Moment Estimation Method*

Elamir and Seheults (2003) developed trimmed L-moment (TL-moments) as generalisation of L-moments, in which they trimmed one smallest and one largest value from the conceptual sample.

TL-moments in terms quantile function can be written as;

$$\lambda_r^{(t_1,t_2)} = \frac{1}{r}\sum_{k=0}^{r-1}(-1)^k \frac{(r + t_1 + t_2)!}{(r + t_1 - k)!\,(t_2 + k)!}\int_0^1 x(F)\,F^{r + t_1 - k - 1}(1 - F)^{t_2 + k}dF \tag{22}$$

If the level of trimming becomes zero (i.e., $t_1 = t_2 = 0.$) the TL- moments reduce to L-moments.

- *The Sample TL Moment is Given by*

$$l_r = \frac{1}{r\binom{n}{r + t_1 + t_2}}\sum_{i=t_1+1}^{n-t_2}\sum_{k=0}^{r-1}(-1)^k \binom{r-1}{k}\binom{i-1}{r+t_1-k-1}\binom{n-i}{k+t_2}x_{i:n} \tag{23}$$

In this work, we focused on trimming of the symmetric cases where $t_1 = 1\ and\ t_2 = 1$

Also, the first four TL-moments (1,1) can be written as follow

$$\lambda_1^{(1,1)} = E(X_{2:3}) = 6\left[\int_0^1 x(F)\,FdF - \int_0^1 x(F)F^2\,dF\right] \tag{24}$$

$$\lambda_2^{(1,1)} = \frac{1}{2}E(X_{3:4} - X_{2:4}) = 6\left[-2\int_0^1 x(F)F^3\,dF + 3\int_0^1 x(F)F^2\,dF - \int_0^1 x(F)\,FdF\right] \tag{25}$$

$$\lambda_3^{(1,1)} = \frac{1}{3}E(X_{4:5} - 2X_{3:5} + X_{2:5}) = \frac{20}{3}\left[-5\int_0^1 x(F)F^4 - 10\int_0^1 x(F)\,F^3dF - 6\int_0^1 x(F)F^2 + \int_0^1 x(F)\,FdF\right] \tag{26}$$

$$\lambda_4^{(1,1)} = \frac{1}{4}E(X_{5:6} - 3X_{4:6} + 3X_{3:6} - X_{2:6}) = \frac{15}{2}\left[-14\int_0^1 x(F)F^5dF + 35\int_0^1 x(F)\,F^4dF - 30\int_0^1 x(F)F^3 + 10\int_0^1 x(F)F^2\,dF - \int_0^1 x(F)\,FdF\right] \tag{27}$$

The sample TL-moment (1,1) can be estimated by setting $t_1 = 1\ and\ t_2 = 1$ in equation (23) above.

The TL- moments (1,1) estimators of the parameters of the GEV distribution are given by

$$\hat{\mu} = l_1^{(1,1)} - \frac{\sigma}{\alpha} - \sigma\Gamma\alpha\,(2.3^{-\alpha} - 3.2^{-\alpha}) \tag{28}$$

$$\hat{\sigma} = \frac{l_2^{(1,1)}}{3\Gamma\sigma(4^{-\alpha} - 2.3^{-\alpha} + 2^{-\alpha})} \tag{29}$$

$$\tau_3 = \frac{\lambda_3^{(1,1)}}{\lambda_2^{(1,1)}} = \frac{\frac{\sigma\Gamma(\alpha+1)}{3\alpha}(20.5^{-\alpha} - 50.4^{-\alpha} + 40.3^{-\alpha} - 10.2^{-\alpha})}{3\Gamma\sigma(4^{-\alpha} - 2.3^{-\alpha} + 2^{-\alpha})} \tag{30}$$

$$\widehat{\tau_4} = \frac{\lambda_4^{(1,1)}}{\lambda_2^{(1,1)}} = \frac{140\left(\frac{4^{\alpha}.3^{\alpha}.2^{\alpha}}{6^{\alpha}}\right) - 420\left(\frac{4^{\alpha}.3^{\alpha}.2^{\alpha}}{6^{\alpha}}\right) + 450(3^{\alpha}.2^{\alpha}) - 200(4^{\alpha}.2^{\alpha}) + 30(4^{\alpha}.3^{\alpha})}{24(3^{\alpha}.2^{\alpha}) - 48(4^{\alpha}.2^{\alpha}) + 24(4^{\alpha}.3^{\alpha})} \tag{31}$$

Where $\tau_3 = \frac{\lambda_3^{(1,1)}}{\lambda_2^{(1,1)}}$ and $\tau_4 = \frac{\lambda_4^{(1,1)}}{\lambda_2^{(1,1)}}$ are the coefficients of the variation, skewness and kurtosis.

- *Proposed New Mixed Estimation Method (MEM)*

In this section, we extend the previous results to estimate the parameters of Generalized extreme value distribution which is call mixed method. The mixed method's central idea is to estimate the location and scale parameters as functions of the shape parameter Morrison and Smith (2002).

In this method, Firstly, location and scale parameters are estimated using TL-moments (1,1). Then, the partial log-likelihood function is maximized to obtain the estimator of shape parameters is substituted in TL-moment (1,1) to get the estimator $\hat{\mu}$ and $\hat{\sigma}$. Details are as follows:

- ✓ Express μ and σ as functions of α using Equations (28) and (29).
- ✓ Substitute $\mu(\alpha)$ and $\sigma(\alpha)$ into the GEV log-likelihood function.
- ✓ Maximize the resulting log-likelihood numerically to obtain $\hat{\alpha}$.
- ✓ Substitute $\hat{\alpha}$ into $\mu(\alpha)$ and $\sigma(\alpha)$.
- ✓ Obtain the mixed estimators

➢ *Mixed Estimation Method for GEV Distribution*

From Equation (29),

$$\hat{\sigma}(\alpha) = \frac{l_2^{(1,1)}}{3\Gamma(\alpha+1)\left(4^{-\alpha} - \left(\frac{2}{3}\right)^{-\alpha} + 2^{-\alpha}\right)}$$

Thus,

$$\sigma(\alpha) = \frac{l_2^{(1,1)}}{3\Gamma(\alpha+1)\left(4^{-\alpha} - \left(\frac{2}{3}\right)^{-\alpha} + 2^{-\alpha}\right)}$$

Substituting $\sigma(\alpha)$into Equation (28),

$$\hat{\mu}(\alpha) = l_1^{(1,1)} - \frac{\sigma(\alpha)}{\alpha} - \sigma(\alpha)\Gamma(\alpha+1)\left[\left(\frac{2}{3}\right)^{-\alpha} - \left(\frac{3}{2}\right)^{-\alpha}\right] \tag{32}$$

Therefore,

$$\mu(\alpha) = l_1^{(1,1)} - \frac{\sigma(\alpha)}{\alpha} - \sigma(\alpha)\Gamma(\alpha+1)\left[\left(\frac{2}{3}\right)^{-\alpha} - \left(\frac{3}{2}\right)^{-\alpha}\right] \tag{33}$$

Hence, both location and scale parameters are now functions of the shape parameter α.

From equation (8), the GEV log-likelihood function is

$$\ell(\mu,\sigma,\alpha) = -n\ln(\sigma) - \left(1 - \frac{1}{\alpha}\right)\sum_{i=1}^{n}\ln\left[1 - \alpha\left(\frac{x_i - \mu}{\sigma}\right)\right] - \sum_{i=1}^{n}\left[1 - \alpha\left(\frac{x_i - \mu}{\sigma}\right)\right]^{1/\alpha}$$

Substituting $\mu = \mu(\alpha),\ \sigma = \sigma(\alpha)$, gives a partial log-likelihood involving only α:

$$\ell(\alpha) = \ell[\mu(\alpha), \sigma(\alpha), \alpha] \tag{34}$$

That is,

$$\ell_p(\alpha) = -n\ln[\sigma(\alpha)] - \left(1 - \frac{1}{\alpha}\right)\sum_{i=1}^{n}\ln\left[1 - \alpha\left(\frac{x_i - \mu(\alpha)}{\sigma(\alpha)}\right)\right] - \sum_{i=1}^{n}\left[1 - \alpha\left(\frac{x_i - \mu(\alpha)}{\sigma(\alpha)}\right)\right]^{1/\alpha} \tag{35}$$

Differentiate the log-likelihood with respect to α:

$$\frac{d\ell(\alpha)}{d\alpha} = 0$$

The resulting equation has no explicit analytical solution; therefore, we used the Newton–Raphson Method numerical optimization technique was employed to obtain $\hat{\alpha}$ which maximizes the log-likelihood function.

To obtain the Mixed Estimators of Location and Scale Parameters, after obtaining $\hat{\alpha}$, substitute it into the TL-moment expressions: $\hat{\sigma}_{MEM} = \sigma(\hat{\alpha})$

That is,

$$\hat{\sigma}_{MEM} = \frac{l_2^{(1,1)}}{3\Gamma(\hat{\alpha}+1)\left(4^{-\hat{\alpha}} - \left(\frac{2}{3}\right)^{-\hat{\alpha}} + 2^{-\hat{\alpha}}\right)} \quad (36)$$

Similarly, $\hat{\mu}_{MEM} = \mu(\hat{\alpha})$

Or

$$\hat{\mu}_{MEM} = l_1^{(1,1)} - \frac{\hat{\sigma}_{MEM}}{\hat{\alpha}} - \hat{\sigma}_{MEM}\Gamma(\hat{\alpha}1)\left[\left(\frac{2}{3}\right)^{-\hat{\alpha}} - \left(\frac{3}{2}\right)^{-\hat{\alpha}}\right] \quad (37)$$

Therefore, the proposed Mixed Method estimators for the GEV parameters are therefore

$$\hat{\theta}_{MEM} = (\hat{\mu}_{MEM}, \hat{\sigma}_{MEM}, \hat{\alpha})$$

Where

$$\hat{\alpha} = \arg\max_{\alpha} \ell(\alpha), \quad (38)$$

$$\hat{\sigma}_{MEM} = \frac{l_2^{(1,1)}}{3\Gamma(\hat{\alpha}+1)\left(4^{-\hat{\alpha}} - \left(\frac{2}{3}\right)^{-\hat{\alpha}} + 2^{-\hat{\alpha}}\right)}$$

And

$$\hat{\mu}_{MEM} = l_1^{(1,1)} - \frac{\hat{\sigma}_{MEM}}{\hat{\alpha}} - \hat{\sigma}_{MEM}\Gamma(\hat{\alpha}+1)\left[\left(\frac{2}{3}\right)^{-\hat{\alpha}} - \left(\frac{3}{2}\right)^{-\hat{\alpha}}\right]$$

III. METHOD SELECTION

Mean square error (MSE) was used to select the best performing estimation methods considered in this work, it can be calculated as below.

$$\text{MSE} = \sum_{i=1}^{n}\left[\hat{F}(x_i) - F(x_i)\right]^2 \quad (39)$$

Where $\hat{F}(x_i)$ is the theoretical CDF at x_i

$$F(x_i) = \frac{i-0.3}{n+0.4} \quad (40)$$

The calculation was performed using computer programming language R and R studio,

- *Return Level and Return Period*

The return period which is also known as recurrence interval, is an estimation of the likelihood of an event, such as rainfall, flood to occur.

The return period (T) is expressed as $\frac{1}{p_e}$ where p_e represents the probability of exceedance

Return levels for 5, 15, 45, 65, 85, and 105 return periods are estimated in this study. Inverting the GEV cumulative distribution, the return level is given by

$$x_T = \mu - \frac{\sigma}{\alpha}\left[1 - \left(-\log\left(1 - \frac{1}{T}\right)\right)^{\alpha}\right] \quad (41)$$

IV. PRESENTATION OF RESULTS

- *Simulation Approach*

Samples were simulated from GEV distribution using Monte Carlo method using R studio and different methods of estimation was compared with proposed method. This comparison was carried out by taking the sample sizes (n= 30, 50 and 200) with different values of shape parameter (-0.3 and 0.3) values of location and scale parameters set to μ=2, σ=2. The mean square error (MSE) was used to obtain the estimation technique which results in the most accurate parameter estimates.

Table 1 Parameter Estimates at Different Sample Sizes Using Simulation at $\alpha = -0.3,\ \mu = 2,\ \sigma = 2$

n	Parameter	MLE	L-MOM	TL-MOM (1,1)	MIXED METHOD
30	$\hat{\mu}$	1.9874	2.0216	2.0082	2.0015
	MSE	(0.3245)	(0.2813)	(0.2462)	(0.1984)
	$\hat{\sigma}$	1.8927	1.9448	1.9734	1.9957
	MSE	(0.3027)	(0.2615)	(0.2238)	(0.1842)
	$\hat{\alpha}$	-0.2514	-0.2743	-0.2871	-0.2965
	MSE	(0.0648)	(0.0532)	(0.0417)	(0.0311)
50	$\hat{\mu}$	2.0113	2.0064	2.0027	2.0008
	MSE	(0.2715)	(0.2321)	(0.1944)	(0.1548)
	$\hat{\sigma}$	1.9358	1.9726	1.9897	2.0018
	MSE	(0.2586)	(0.2217)	(0.1814)	(0.1462)
	$\hat{\alpha}$	-0.2711	-0.2846	-0.2935	-0.2981
	MSE	(0.0483)	(0.0397)	(0.0316)	(0.0224)
200	$\hat{\mu}$	2.0048	2.0025	2.0016	2.0001
	MSE	(0.1326)	(0.1185)	(0.0962)	(0.0714)
	$\hat{\sigma}$	1.9813	1.9914	1.9978	2.0006
	MSE	(0.1284)	(0.1127)	(0.0895)	(0.0658)
	$\hat{\alpha}$	-0.2895	-0.2941	-0.2972	-0.2996
	MSE	(0.0196)	(0.0162)	(0.0118)	(0.0081)

- *Interpretation:*

The results from Table 1 indicated that the proposed Mixed Method produced estimates that were consistently closer to the true parameter values than the existing methods. Furthermore, the Mixed Method has the smallest MSEs for the location, scale, and shape parameters across all the sample sizes considered. As the sample size increased from 30 to 200, the MSE values decreased for all methods, demonstrating improved estimation accuracy and consistency.

Table 2 Parameter Estimates at Different Sample Sizes Using Simulation at $\alpha = 0.3,\ \mu = 2,\ \sigma = 2$

n	Parameter	MLE	L-MOM	TL-MOM (1,1)	MIXED METHOD
30	$\hat{\mu}$	2.0479	2.0161	2.0189	1.9938
	MSE	(0.3273)	(0.3016)	(0.3033)	(0.2140)
	$\hat{\sigma}$	1.7687	1.7929	1.8461	1.4932
	MSE	(0.3078)	(0.2838)	(0.2950)	(0.2039)
	$\hat{\alpha}$	0.2746	0.2812	0.2875	0.2957
	MSE	(0.0624)	(0.0547)	(0.0486)	(0.0332)
50	$\hat{\mu}$	2.2246	2.1936	2.1917	2.1322
	MSE	(0.2964)	(0.2113)	(0.2817)	(0.2277)
	$\hat{\sigma}$	1.9749	1.9636	1.8888	1.5978
	MSE	(0.2749)	(0.2012)	(0.2773)	(0.2451)
	$\hat{\alpha}$	0.2863	0.2911	0.2948	0.2984
	MSE	(0.0418)	(0.0364)	(0.0287)	(0.0196)
200	$\hat{\mu}$	1.9119	1.9141	1.9191	1.9752
	MSE	(0.2328)	(0.2100)	(0.2589)	(0.1592)
	$\hat{\sigma}$	1.9297	1.9404	1.9429	1.7607
	MSE	(0.2171)	(0.2016)	(0.2583)	(0.1521)
	$\hat{\alpha}$	0.2954	0.2967	0.2979	0.2997
	MSE	(0.0128)	(0.0107)	(0.0093)	(0.0058)

- *Interpretation:*

The results in Table 2 shows that the proposed Mixed Method produces estimates that are closer to the true parameter values and generally yields smaller MSEs than the other methods. This indicates that the Mixed Method provides more efficient estimation of the GEV parameters, particularly for moderate and large sample sizes.

➢ *Application to Real Life Data*

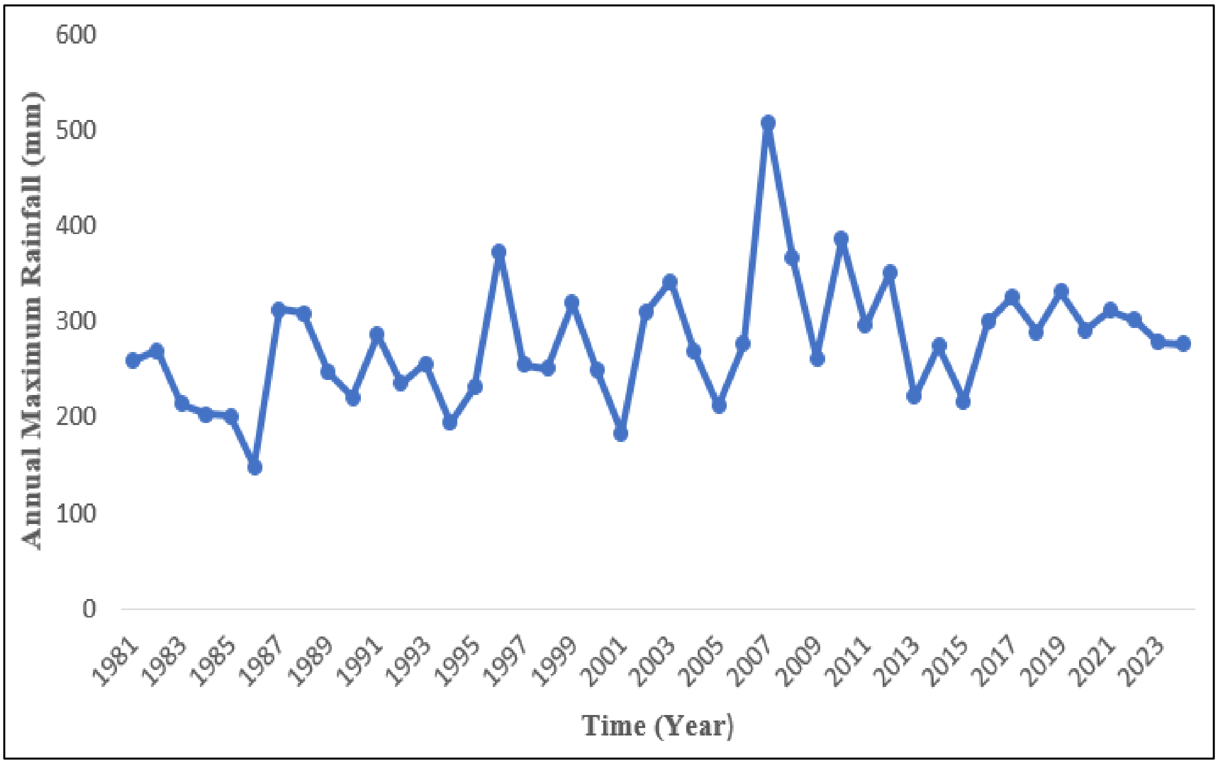

Fig 1 Time Plot Showing Annual Maximum Rainfall Distribution Across Ogun State Between 1981 and 2024.

- *Interpretation:*

Figure 1 presents a time plot showing annual maximum rainfall distribution of Ogun state. The graph shows the systematic system of the series or data over a period of time. Time shows where there is extreme annual maximum rainfall.

Table 4 Parameter Estimates of GEV by Several Methods Using Ogun State Rainfall Data

State	Parameter estimates	MLE	L-MOM	TL-MOM (1,1)	METL (1,1)
Ogun	$\hat{u}$	247.16	246.83	249.12	246.51
	$\hat{\sigma}$	62.42	63.78	64.86	54.89
	$\hat{\alpha}$	-0.081	0.089	0.202	-0.061
MSE		0.1503	0.0008	0.0027	5.666e-06

- *Interpretation:*

Table 4. shows the parameter estimates of GEV by the MLE, L-moment, TL-moments and MEM using Ogun state rainfall data. The results show that the MEM gave the least mean square error (MSE) value, this also indicates that MEM is the most efficient estimator for the Ogun state rainfall data and it outperforms all the other three existing standard methods.

Table 5 Prediction of Return Level for Different Return Periods in Ogun State

Return Period	MLE	L-MOM	TL-MOM (1,1)	MEM
T=5	347	336	333	334
T=15	475	424	401	446
T=45	525	452	420	490
T=65	557	468	431	518
T=85	581	480	438	539
T=105	600	489	444	556

- *Interpretation:*

Table 5 shows that the return level estimate by different methods of estimation increases as the return period increases. The rainfall amount which exceeds the maximum rainfall amount (508.2) of the observation period started appearing at T = 45,65,85 to 105.

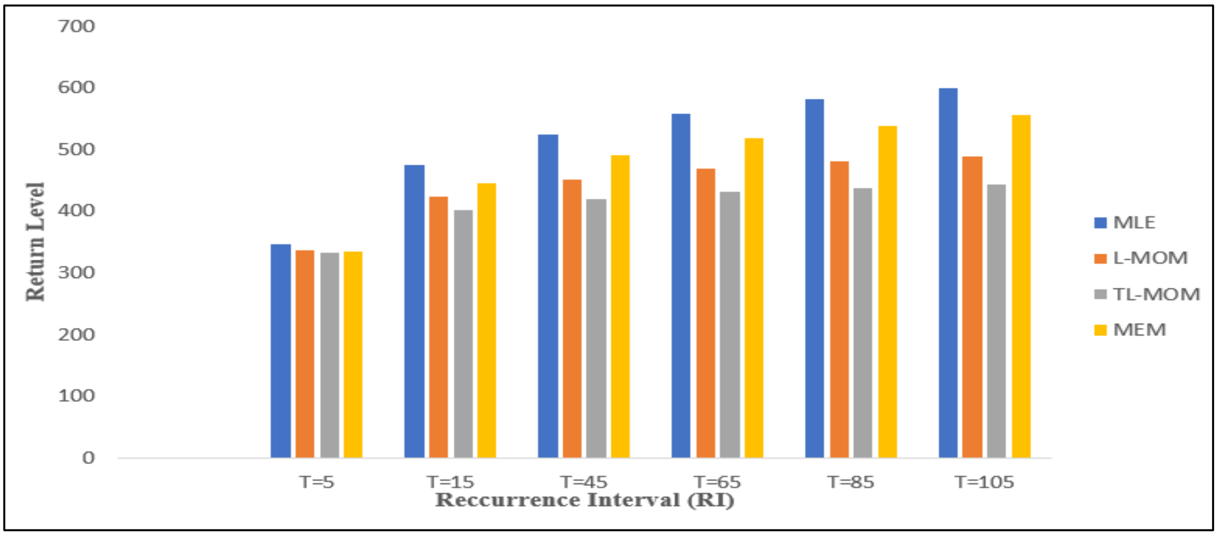

Fig 2 Multiple Bar Chart Showing Rainfall Recurrence Interval for Different Return Period in Ogun State.

- *Interpretation:*

Figure 2 presents recurrence interval of Ogun State annual maximum rainfall using the newly developed methods and three standard methods.

V. DISCUSSION

The simulation results presented in Tables 1 and 2 show that the proposed Mixed Estimation Method (MEM) consistently outperformed the Maximum Likelihood Estimation (MLE), L-Moments (L-MOM), and TL-Moments (1,1) methods in estimating the parameters of the GEV distribution. For both negative ($\alpha = -0.3$) and positive ($\alpha = 0.3$) shape parameter values, the MEM produced parameter estimates that were closer to the true values and yielded the smallest Mean Squared Errors (MSEs) across the sample sizes considered. Furthermore, the MSE values decreased as the sample size increased from 30 to 200, indicating improved estimation accuracy and consistency of the estimators.

The application of the methods to the annual maximum rainfall data of Ogun State further shows that the proposed MEM is more efficient than other methods considered in this work. Similarly, the return level analysis showed that estimated rainfall magnitudes increased with increasing return periods, which is consistent with the theoretical behaviour of extreme events. These findings suggest that the proposed MEM is a robust and efficient estimation technique for modelling extreme hydrological events using the GEV distribution.

VI. CONCLUSION

This study developed a Mixed Estimation Method (MEM) for estimating the parameters of the Generalized Extreme Value (GEV) distribution by combining the robustness of TL-Moments with the efficiency of Maximum Likelihood Estimation. The simulation results showed that the proposed method consistently produced more accurate parameter estimates and smaller Mean Squared Errors than the existing MLE, L-Moment, and TL-Moment methods across different sample sizes and shape parameter values. The application to annual maximum rainfall data in Ogun State further shows that the proposed method outperforms the existing methods. Therefore, the proposed MEM is an efficient and robust alternative for modelling extreme events and can serve as a valuable tool for hydrological risk assessment, flood forecasting, and environmental decision-making.

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