International Journal of Scientific Research and Modern Technology (IJSRMT)
Volume 4, Issue 3, 2025
DOI: https://doi.org/10.38124/ijsrmt.v4i3.364
ijsrmt.com

Modification Parameter Estimation of the Seasonal Autoregressive Moving Average Process in the Present of Arm (1, 1)

Ibrahim Maihaja[1]; Jibril Lawal[2]

[1] Department of Mathematics and Statistics, Abdu Gusau Polytechnic, Talata Mafara, Zamfara, Nigeria
[2] Department of Mathematical Sciences, Federal University Gusau, Zamfara, Nigeria

Publication Date: 2025/04/22

Abstract

Error was known to be persistent in the measurement of parameters. This research work investigated seasonal autoregressive integrated moving averages with time series error patterns ($SARIMA$-E). The study enhances the minimized effect and improves the model resolutions. $SARIMA$-E with auto-regressive moving average error $ARMA(1,1)$ is considered using the maximum likelihood method, iteration procedure, and chi-square test. However, the result shows that the optimum model is very significant. Furthermore, the forecast performance measurement and properties of errors with different values of parameters were investigated and analyzed. The test of the seasonal unit root was observed in both cases and the simulations were conducted using $R-$statistical software and monthly Temperature real data obtained for the period between $1988\,to\,2022$ in Zamfara was equally used to validate the results. Finally, the research finding depicts that the result obtained was too close to the parameter values of the model analytical results obtained and presented. It was suggested that these results would be useful in predicting the stability of the system.

Keywords: *$SARIMA$, ARMA (1,1), Parameter Estimation, Auto-Covariance Function, Statistical Software, Error.*

I. INTRODUCTION

This research work is all about modification parameter estimation and modeling error process in $SARIMA$ to follow an Autoregressive error process unlike the usual independent identically distributed error (NID) with mean zero, a constant variance σ^2, zero auto covariance, and zero autocorrelation. The current error processes are predictable in that they follow a time series pattern. This is necessary to minimize errors in the modeling procedure and to improve the precision of the estimator of parameters, the most recent work on this time series included the following.

Maihaja et al. (2023) studied the forecast performance measurement and properties of error with different values. They carried out the Test of seasonal unit roots and the simulation using the real data of Zamfara's monthly Rainfall from $1988\,to\,2022$ to validate the analytical results with $R-$statistical software. The result showed a significant p-value and their formulated model provides a generalization and more flexible specification than the existing models of $AR(1)$ error and $ARMA(1,1)$ error in fitting time series processes in the presence of errors.

Komolafe et al. (2019) investigated an Integrated Moving Average (IMA) model with a transition matrix for error. They applied Kolmogorov Smirnov test statistic, the autocovariance function, the maximum likelihood techniques, and the Raphson iterative approach as their instruments. However, data sets on interest rates in the United States and Nigeria were used to demonstrate the application of the proposed model. Their findings show that the model for fitting time series processes in the presence of error offered a more flexible specification and generalization than the current models of $ARMA$ and AR.

Rudelson and Zhou (2017) studied the convergence rates of gradient descent methods for solving the no-convex programs and concluded that the composite gradient descent approach is guaranteed to converge at a geometric rate to a neighborhood of the global minima. The result suggests connections between measure phenomena concentration in random matrix theory and statistical and computing efficiency.

Valipour (2012) attempts to forecast the inflow of the Dez dam reservoir by using $ARMA$ and $ARIMA$ models while increasing the number of parameters in $ARMA$ and $ARIMA$ models, the polynomial was derived with four and six parameters, respectively, to forecast the inflow. By comparing the root mean square error of the models, it was determined that the $ARIMA$ model can forecast inflow to the Dez reservoir from 12 months ago with a lower error than the $ARMA$ model.

Madansky (1959) considered the situation where U and V are related by, $Y = \alpha + \beta x$ where α and β are unknown for fitting of straight lines when both variables are subjected to errors they observed $u = U + x$ and $v = V + y$ assume that $Ex = Ey = 0$ and that the errors (x and y) are uncorrelated with the true values (U and V). They surveyed and commented on the solutions to the problem of obtaining consistent estimates of α and β from a sample of (u,v)'s. More recently, detailed work could be found in the following studies (Ajobo et al., 2024; Maihaja et al., 2023; Mamuda et al., 2021; Neusser, 2016; Schneeweiss and Shalabh, 2007).

Ayodeji (2016) developed a model to investigate the long swings hypothesis in currencies that exhibit ak a state $(k \geq 2)$ pattern, the model was used on Euros, British pounds, Japanese yen, and Nigerian naira. However, the result shows the presence of asymmetric swings in the naira and yen and also long swings in Euros and pounds.

Eni, (2013), used the simulation method for parameter estimation of the first-order IMA model in the presence of $ARMA$ (1, 1) errors. It revealed that the method was very close to the estimated true parameters of the error process (see also, Eni and Mahmud, 2008).

This research paper aims to estimate the parameters of the $SARIMA$ Model by examining the error process as $ARMA$ (1,1) to achieve the following objectives, formulate a $SARIMA$ model corrupted with $ARMA$ (1,1) error process, estimate the parameters of the formulated model using the maximum likelihood method approach, to examine the properties of error pattern and variation with different values of the parameters, to test the seasonal unit root on simulated data and investigate the forecast Performance Measures.

Furthermore, this research paper tends to help researchers, decision-makers, and above all to add value to the existing literature on time series models with corrupted error processes, and their applications.

II. MATERIALS AND METHOD

A. *Sarima Model*

The seasonal auto-regressive integrated moving average ($SARIMA$) model is the generalization of the well-known Box-Jenkins model to accommodate data with both seasonal and non-seasonal features. The $ARMA$ model which is known to be a combination of the auto-regressive (AR) and moving average (MA) models utilizes past information of a given series in other to predict the future. The AR part of the model deals with the past observation of the series while the MA part deals with the past error of the series. The $SARIMA$ model is a version of the common $ARIMA$ model which also incorporates a seasonal part. Thus, the general $SARIMA$ model can be expressed (Box and Jenkins, 2015; Hamilton, 1994; Shumway and Stoffer, 2006).

$$\phi_p(L)\Phi_P(\mathrm{L}^S)\nabla^d\nabla_s^D Y_t = \theta_q(L)\Theta_Q(L^S)\varepsilon_t \quad (1)$$

Using Lag operators, the Autoregressive (AR) and Moving Average (MA) polynomials are defined respectively as:

$$\phi_p(\mathrm{L}) = 1 - \phi_1 L - \phi_2 L^2 - \ldots - \phi_p \mathrm{L}^P \quad (2)$$

$$\theta_q(L) = 1 + \theta_1 L + \theta_2 L^2 + \ldots + \theta^q L^q \quad (3)$$

The seasonal AR and MA polynomials are defined respectively as follows:

$$\Phi_P(L) = 1 - \Phi_1 L^S - \Phi_2 L^{2S} - \ldots - \Phi_P L^{PS} \quad (4)$$

$$\Theta_Q(L) = 1 + \Theta_1 L + \Theta_2 L^{2S} + \ldots + \Theta_Q L^{QS} \quad (5)$$

Where s is the seasonal level (s =4 for quarterly data), ∇^d the order of integration and ∇^{DS} the order of seasonal integration, and ε_t is a white noise process (Brock well and Davis, 2016). In an autoregressive model, Y_t is a linear combination of the p most recent values (Cryer and Chan, 2008).

Rather than using past values, the MA part of the model includes past error. The consequence is that any shock on Y_t will gradually fade off in the case of the AR model, but will do so abruptly in the case of MA model. $SARIMA(p,d,q)\times(P,D,Q)^S$ with error process,

$$(1-L)X_t = e_t + (\theta - 1)e_{t-1} \quad (6)$$

$$(1-\mathrm{L})(1-\mathrm{L}^{12})\,\mathrm{y}_t = (1-\theta L)(1-\Theta L^{12})\varepsilon_t \quad (7)$$

B. *Parameter Estimation*

Parameter estimation is the second step of the Box-Jenkins methodology. The estimation method used in the $SARIMA$ model is maximum likelihood. Simulation studies to compare unconditional least squares, conditional least squares, and maximum likelihood for $ARMA$ models have been conducted by (Ansley and New Bold, 1980; Dent and Min, 1978). These simulations suggest that although unconditional least squares and conditional least squares are adequate approximations of the maximum likelihood for large sample sizes, maximum likelihood is preferred for small and moderate sample sizes (Box and Jenkins, 2015).

C. Forecast Performance Measures

The forecast performance measures or forecast performance metrics. Now, in applying a particular model to some real or simulated time series to generate forecasts, we first divide the raw data into two parts:

i. *Consider SARIMA model corrupted with*
ii.

$$w_t = (1-\phi L)(1-\lambda L^{12}) \in_t + (1-L)(1-L^{12}) b_t \quad (8)$$

$$b_t = \frac{(1+\alpha_2 L)\mathrm{e}_t}{(1-\alpha_1 L)} \quad (9)$$

$$w_t = (1-\phi L)(1-\lambda L^{12})\varepsilon_t + (1-L)(1-L^{12})$$

Let

$$z_t = (1-\alpha_1 L) w_t \quad (10)$$

Grouping

$$u_t = \varepsilon_t + \ell_t \quad (11)$$

$$\Omega_1 u_{t-1} = -(\phi + \alpha_1)\varepsilon_{t-1} - (1-\alpha_2)\ell_{t-1} \quad (12)$$

$$\Omega_2 u_{t-2} = \alpha_1 \varepsilon_{t-2} - \alpha_{2\ell_{t-2}} \quad (13)$$

$$\Omega_3 u_{t-12} = -\lambda \varepsilon_{t-12} - \ell_{t-12} \quad (14)$$

$$\Omega_4 u_{t-13} = (\lambda\phi + \alpha_1\lambda)\varepsilon_{t-13} + (1+\alpha_2)\ell_{t-13} \quad (15)$$

$$\Omega_5 u_{t-14} = \alpha_2 \ell_{t-14} - \alpha_1 \lambda \phi \varepsilon_{t-14} \quad (16)$$

$$z_t = u_t - \Omega_1 u_{t-1} + \Omega_2 u_{t-2} - \Omega_3 u_{t-12} + \Omega_4 u_{t-13} + \Omega_5 u_{t-14} \quad (17)$$

The developed model of equation (17) was $SARIMA(0,0,2)(0,0,1)_{12}$

Multiply equation (17) by z_t and take expectation

$$v_o = \sigma^2 - \Omega_1\sigma^2 + \Omega_2\sigma^2 - \Omega_3\sigma^2 + \Omega_4\sigma^2 + \Omega_5\sigma^2 \quad (18)$$

Multiply equation (17) by z_{t-1} and take expectation

$$v_1 = (\Omega_4\Omega_5 - \Omega_3\Omega_4 - \Omega_1\Omega_2 - \Omega_1)\sigma^2 \quad (19)$$

III. RESULTS AND DISCUSSIONS

Table 1 The Results of $SARIMA(0,0,2)\ (0,0,1)_{12}$ Corrupted with $ARMA(1,1)$ Error Process

Process	***Estimate***	***Stan. E***	***Z – value***	***P – value***
Ma 1	0.4980	0.0149	33.4228	$2.51836e^{-230}$
Ma 2	0.3519	0.0516	22.5577	$6.792548e^{-103}$
Sma 1	- 0. 0313	0.0141	-2. 2199	0.0264
Mean	-0.2004	0.0302	-6.6358	$3.35939e^{-12}$

Sigma^2 estimated as 0.8469
log likelihood = - 6677.15
AIC = 13364 **AICC**= 13364.31 **BIC** = 13396.89

The result in Table 1 showed $SARIMA$ $(0,0,2)(0,0,1)_{12}$ this implies that when the error process switches to $ARMA$ (1,1) it is the moving average of order two and the seasonal moving average of order one with a period of twelve months. The estimation of all the parameters showed very significance results.

Table 2 Results of Forecast Performing and Properties of Error and Variation with Different Values of (1, 1) Error Values

	ME	***RMSE***	***MAE***	***MPE***	***MAPE***	***MASE***	***ACFI***
Training set	-- $8.9480e^{-06}$	0.9199	0.7328	127.2838	156.1934	0.6918	0.0010

The results in the table 2 showed the forecast performing measurement or training set and properties of error with difference values when an error process switch to $ARMA(1,1)$.

Table 3 Estimates at Each Iteration

Iteration	***SSE***	***Parameters***			
0	16151.1	0.100	0.100	0.100	65.774
1	12076.0	-0.050	0.094	0.093	65.765
2	8924.6	-0.200	0.032	0.076	65.752
3	6778.4	-0.350	-0.084	0.037	65.732
4	5548.9	-0.500	-0.233	-0.036	65.704

5	5104.3	-0.636 -0.383 -0.181 65.667
6	5070.0	-0.588 -0.337 -0.249 65.688
7	5066.8	-0.607 -0.370 -0.248 65.697
8	5065.3	-0.587 -0.355 -0.250 65.699
9	5064.8	-0.594 -0.368 -0.250 65.699
10	5064.6	-0.586 -0.362 -0.251 65.699
11	5064.5	-0.589 -0.367 -0.251 65.699
12	5064.4	-0.586 -0.365 -0.251 65.700
13	5064.4	-0.587 -0.367 -0.251 65.700
14	5064.4	-0.586 -0.366 -0.251 65.700
15	5064.4	-0.586 -0.367 -0.252 65.700
16	5064.4	-0.586 -0.367 -0.252 65.700

Relative change in each estimate is less than 0.0010.

Table 3 above depicts the results of the parameter estimate at each iteration for the *SARIMA* model corrupted with *ARMA* (1,1) error process.

Table 4 Final Estimates of Parameters

Type	***Coeff.***	***SE Coeff***	***T***	***P***
MA 1	-0.5858	0.0788	-7.44	0.000
MA 2	-0.3666	0.0786	-4.67	0.000
SMA 12	-0.2515	0.0825	-3.05	0.003
Constant	65.700	1.221	53.80	0.000
Mean	65.700	1.221		

Number of observations: 144

Residuals:

SS = 5007.14 (back forecasts excluded)

MS = 35.77 **DF** = 140

Table 4 depicts the results of the final parameter estimate for *SARIMA* model corrupted with *ARMA*(1,1) error process.

Table 5 Chi-Square Statistic

Modified Box – Pierce (Ljung-Box) Chi – square statitics				
Lag	12	24	36	48
Chi – square	81.9	98.4	115.1	143.8
DF	8	20	32	44
P – Value	0.000	0.000	0.000	0.000

The results in Table 5 are the Chi-square statistic for the *SARIMA* model corrupted with *ARMA*(1,1) error process at lag 12, 24, 36, and 48 respectively.

IV. SIMULATIONS

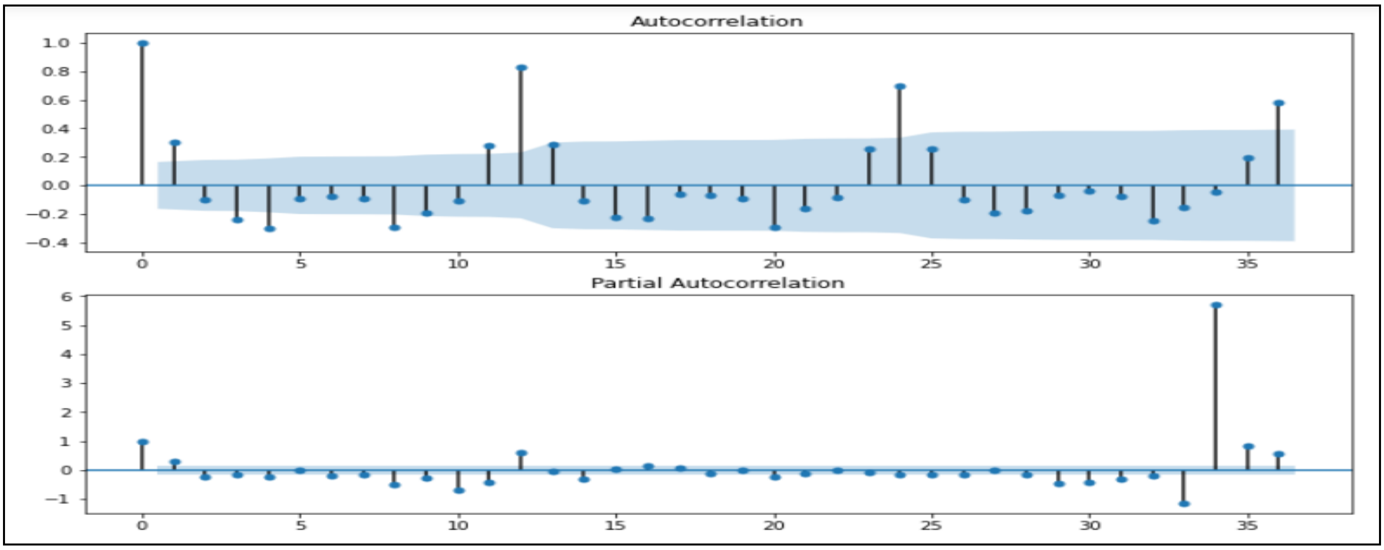

Fig 1 Plot of *ACF* and *PACF* at Lags.

The *ACF* plot in figure 1 shows the correlation of the series with its lagged values, it describes how present value of the series related with its past and consider seasonal effect with upper confidence interval. *PACF* plot shows the correlation of the residuals in the series and lags.

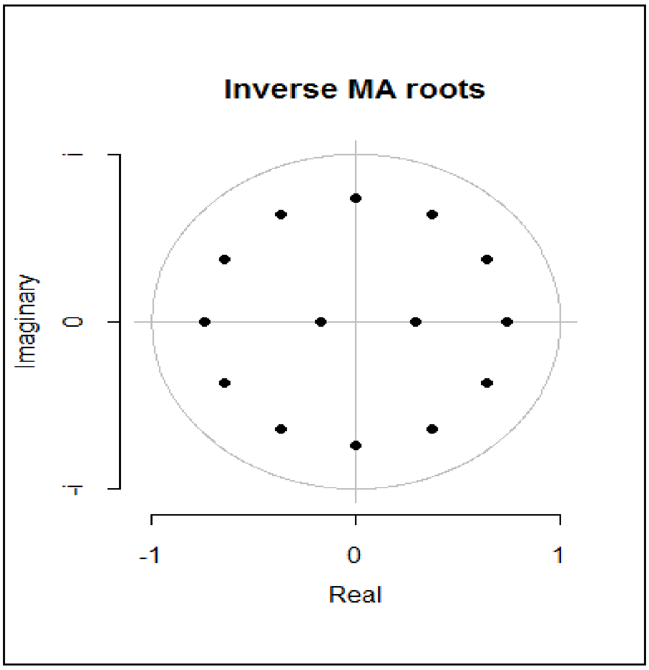

Fig 2 Plot for Seasonal Unit Root of $ARMA(1, 1)$ Inside the Unit Circle.

The plot in figure 2 is pictorial of *SARIMA* model for the period of twelve (12) months with a double *MA* root i.e. *SARIMA* $(0.0.2)(0,0,1)^{s=12}$ the twelve circle point represent the seasonal effects while the double inner point represent the(2) root for the Monthly Temperature of Zamfara from 1988 *to* 2022.

V. CONCLUSION

Modified *SARIMA* models have been corporated with $AR(1)$ and $ARMA(1,1)$ errors. The study employs autocovariance function, maximum likelihood, and Chi-Statistic to estimate the parameters of those models with the aid of iterative techniques, Minitab software, Python, and $R-$software. Furthermore, the measurement of the inaccuracy of forecast performance was carried out using both simulated and actual data. It was observed that all unit-roots fell inside the unit circle and were in compliance with the results of the seasonal unit root test on the data presented.

RECOMMENDATION

It was recommended that conditional Heteroskedasticity (*ARCH*) shall be considered for volatility. Also, collection of more data would be needed for future improve results. Again, *SARIMA* models corrupted with *ARMA* (1,1) would certainly enhance research if other version of time series like Generalized Autoregressive Autoregressive and Conditional Heteroskedasticity were employed.

REFERENCES

[1]. Ajobo S., Alaba O.O., and Zaenal A. (2024). Generalised space-time seasonal autoregressive integrated moving average seemingly unrelated regression modelling of seasonal and non-stationary data, Scientific African.

[2]. https://doi.org/10.1016/j.sciaf.2024.e02189.

[3]. Ansley C. F, and New bold P.(1980). Finite Sample Properties of Estimators for Autoregressive Moving Average Models, *Journal of Econometrics*, 13(2):159-183.

[4]. Ayodeji I. O. (2016). A three State Markov-Modulated Switching Model for Exchange Rates,'' *Journal of Applied mathematics*, 2016, 1 – 9.

[5]. Box G., and Jenkins G. (2015). Times Series Analysis*:* Forecasting and Control, San Francisco, CA: Holden Day.

[6]. Brock well P. J., and Davis R. A. (2016). Introduction to Time Series and Forecasting, *Springer, International Publishing*, Cham, 3rd edition.

[7]. Cryer J. D., and Chan K. (2008). Time Series Analysis: with Applications in R,'' *Springer*, New York, 2nd edition.

[8]. Dent W, and Min A. (1978). A Monte Carlo Study of Autoregressive Integrated Moving Average Processes,'' *Journal of Econometrics*, 7(1):23-55.

[9]. Eni D. (2013). Parameter Estimation of First Order IMA Model in the Presence of ARMA (1,1) Errors, *Research journal in Engineering and applied Science*, 2(4), 246 – 50.

[10]. Eni D., and Mahmud S. A. (2008). On the Parameter Estimation of First Order IMA Model Corrupted with AR (1) Errors,''*Global Journal of Pure and applied Science*, 14(1), 115 – 20.

[11]. Hamilton D. J, (1994). *Time Series Analysis.* Princeton Univ. Press, Princeton, New Jersey.

[12]. Komolafe S. A., Obi lade T. O, Ayodeji I. O.,, and Babalola A. R. (2019). Development of First Order Integrated Moving Average Model Corrupted with a Markov Modulated Convex Combination of Autoregressive Moving Average Errors, *Journal of statistical theory*, 3:1, 48 – 58.

[13]. Madansky A. (1959). The Fitting of Straight Lines when Both Variable are Subject to Error'' *Journal of the American Statistical Association*, 54, 173 – 205.

[14]. Maihaja I., Wasiu A. B., & Onwuka I. G. (2023). Parameter Estimated of Seasonal Autoregressive Integrated Moving Average Model with AR(1) Error Process,'' Asian J. Prob. Stat., 25(1), pp.100-111, DOI:10.9734/ajpas/2023/v25i1543.

[15]. Mamudu L., Yahaya A., Dan'azumi S. (2021). Application of Seasonal Autoregressive Integrated Moving Average (SARIMA) For Flows of River Kaduna, ''Nigerian Journal of Engineering, 28(2), ISSN (print): 0794 – 4756, ISSN(online):2705-3954.

[16]. Neusser K. (2016). Time Series Econometrics,'' Springer, New York.
[17]. Rudelson M., and Zhou S. (2017). Errors in Variables Models with Dependent Measurement *Electronic Journal of Statistics*, 11, 1699 – 1797.
[18]. Schneeweiss H., and Shalabh H. (2007). On the Estimation of the Linear Relation when the Error Variances are known,'' Computational Statistics & Data Analysis, 52, 1143 – 1148.
[19]. Shumway R. H., and Stoffer D. S. (2006). *Time Series Analysis and Its Applications: with R Examples*.'' Springer, New York, 2nd edition.
[20]. Maihaja I., Babayemi A. W. and G. I. Onwuka (2023). Development Of Seasonal Autoregressive Integrated Moving Average Model With Error Process, Int. J. Adv. Res. 11(09), 1198-1205, ISSN: 2320-5407. DOI: 10.21474/IJAR01/17635.
[21]. Valipour M. (2012). Parameters Estimate of Autoregressive Moving Average and Autoregressive Integrated Moving Average Models and Compare Their Ability for Inflow Forecasting, Journal of Mathematics and Statistics 8 (3): 330-338, ISSN 1549-3644.